The algorithmic Weak Stability Boundary in Earth-to-Moon transfer orbit design: applicability of solutions in the associated stable set and classification of stability transitions

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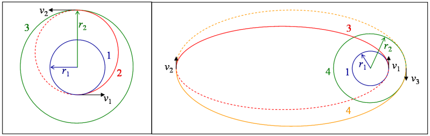
Summary

Motivation and Context

- Traditional techniques in astronautics



Hohmann transfer *Bi-elliptic transfer*

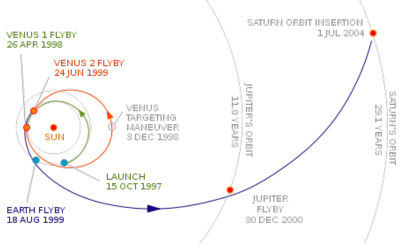
New challenges require new techniques!

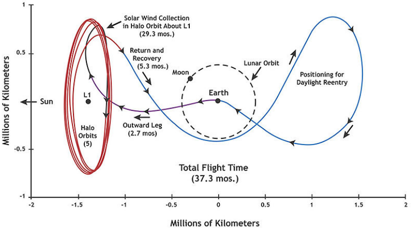
- Patched conics: gravity assisted maneuvers to save fuel (swingby or gravitational slingshot)

  E.g.: Cassini (Oct 15, 1997): Saturn multi-moon orbiter

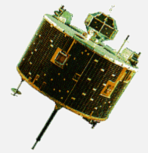
- Take advantage of the fundamental dynamical structure of more realistic (N-body) models!

  E.g.: Genesis Mission (Aug 8, 2001): Approximate heteroclinic return orbit to bring back to Earth solar wind particles





Motivation and Context

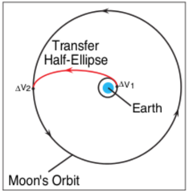


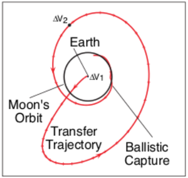
- Space mission projects based on many-body dynamics: particularly Sun-Earth-Moon-Sc.
- WSB concept proposed heuristically by E. Belbruno (1987) related to Earth-Moon transfers with ballistic capture.
- Employed successfully in the rescue of the Japanese spacecraft Hiten in 1991 (E. Belbruno, J. Miller (1990)).

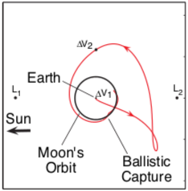
"Regions in the phase space where the perturbative effects of the Earth-Moon-Sun acting on the spacecraft tend to balance".
E. Belbruno, J. Miller (1993)

"A location near the Moon where the spacecraft lies in the transition between ballistic capture and ejection".
E. Belbruno, F. Topputo, M. Gidea (2008)

But... Precise definition? Why does it work? How to find WSB trajectories?







Motivation and Context

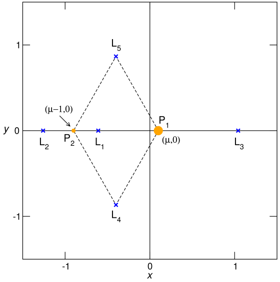
References

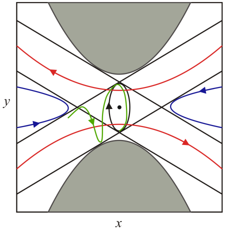
- P.A. Sousa Silva, M.O. Terra. **Diversity and validity of stable-unstable transitions in the algorithmic weak stability boundary**. Celestial Mechanics and Dynamical Astronomy, vol 113, 4, p. 453-478 (2012)
- P.A. Sousa Silva, M.O. Terra. **Applicability and dynamical characterization of the associated sets of the algorithmic weak stability boundary in the lunar sphere of influence**. Celestial Mechanics and Dynamical Astronomy, vol 113, 2, p. 141-168 (2012)
- P.A. Sousa Silva, M.O. Terra. **Dynamical properties of the weak stability boundary and associated sets**. Journal of Physics. Conference Series (Online), v. 246, 012007 (2010)

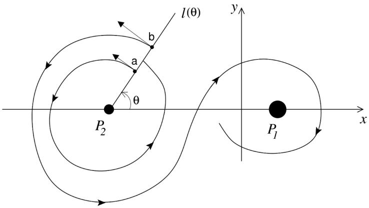
Weak Stability Boundary Algorithmically Defined

Procedure

- Set framework: The [Planar Circular] Restricted Three-body Problem
- Define initial conditions
- Perform stability classification
- Extract stability boundary







(E. Belbruno (2004); F. García, G. Gómez (2007))

Grid dependence! Integration time dependence!

Some definitions

Definition (Ballistic capture: analytic concept - see E. Belbruno (2004)*)

P_3 is ballistically captured by P_2 at time $t = t_c$ if, for a solution $\varphi(t) = (x(t), y(t), \dot{x}(t), \dot{y}(t))$ of the R3BP, $h_K(\varphi(t_c)) \leq 0$, where h_K is the two-body energy of P_3 with respect to P_2.

* Not unique! See W. Koon, M. Lo, J. Marsden, S. Ross (2001); J. Marsden, S. Ross (2005).

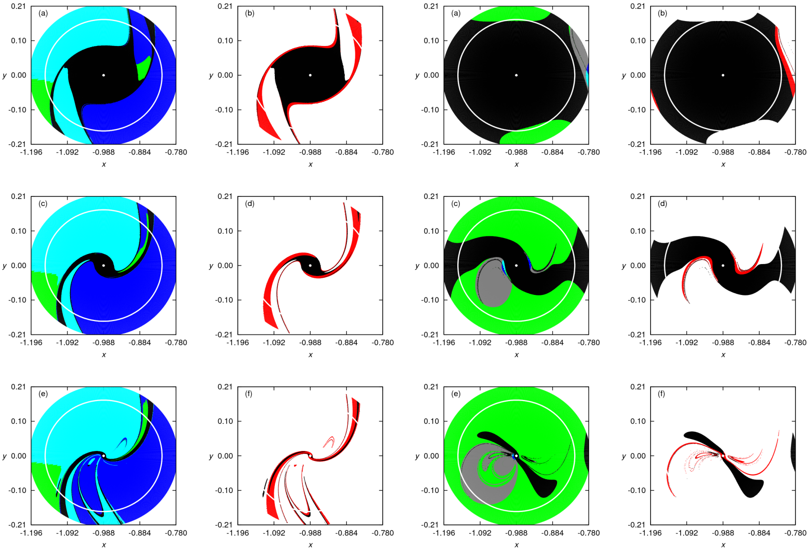
Definition (Stability)

The motion of P_3 is said to be **stable** *if after leaving $l(\theta)$ it makes a full cycle about P_2 without going around P_1 and returns to $l(\theta)$ with $h_K < 0$. The motion is* **unstable** *otherwise.*

Definition (Algorithmic WSB)

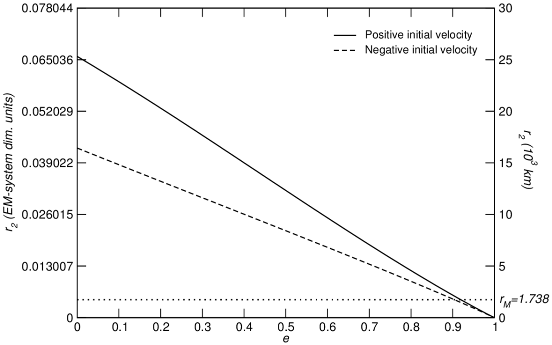
The Weak Stability Boundary is given by the set $\partial\mathcal{W} = \{r^* | \theta \in [0, 2\pi), e \in [0, 1)\}$, where $r^*(\theta, e)$ are the points along the radial line $l(\theta)$ for which there is a change of stability. The subset obtained by fixing the eccentricity e of the osculating ellipse is $\partial\mathcal{W}^e = \{r^* | \theta \in [0, 2\pi), e = \text{constant}\}$.

Implementation of the algorithmic WSB



Preliminary checks: the energy

High excentricity needed to allow low capture orbits!



$$C(r_2,\theta,e) = (1-\mu) + \frac{2\mu}{r_2} - 2(1-\mu)r_2\cos(\theta) + r_2^2 + \frac{2(1-\mu)}{\sqrt{1-2r_2\cos(\theta)+r_2^2}} - \left[r_2 \mp \sqrt{\frac{\mu(1+e)}{r_2}}\right]^2$$

Energy gap between + and - sets of initial conditions: $\Delta C(r_2,\theta,e) = 4\sqrt{\mu(1+e)r_2}$.

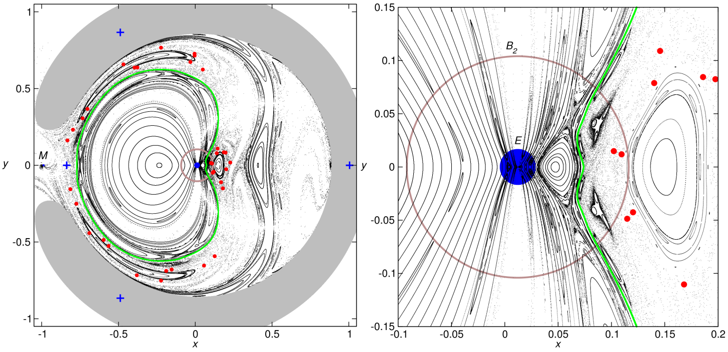
Inner Transfers: within the restricted three-body problem

- Scheme to design low energy "periodic" Earth-to-Moon transfers. C. Conley (1968)

  (i) *the cost per cycle should be as small as is practical;*
  (ii) *control and stability problems should be as easy as possible;*
  (iii) *as much flexibility should be build into the scheme as possible.*

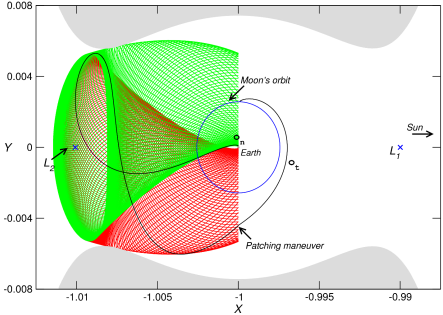
But...

- It is impossible to go from a region arbitrarily close to the Moon to a region arbitrarily close to the Earth due to shielding invariant torus. R. McGehee (1969)



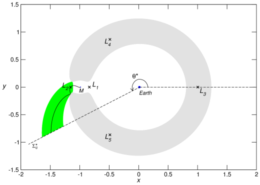
Outer transfers

- **Four body models are required to obtain assist by the Sun!**
- **Patched Three-Body approach***
  - Sun-Earth-SC ($SE \Leftrightarrow \odot$) + Earth-Moon-SC ($EM \Leftrightarrow \oplus$).
  - Non-transit orbit o_n associated to $L_1^{\odot}$ or $L_2^{\odot}$ + Transit orbit o_t associated to $L_2^{\oplus}$.
  - Total energy: Δv_1 to leave o_i + Δv_2 at patching point + Δv_3 to enter o_f.



SE setup: x-y projection of the inner branches $W^s_{\odot}$ (green) and $W^u_{\odot}$ (red) of $\Gamma_2^{\odot}$

$C^{\odot} = 3.00080369$



Corresponding EM setup: $W^s_{\oplus}$ (green) of $\Gamma_2^{\oplus}$

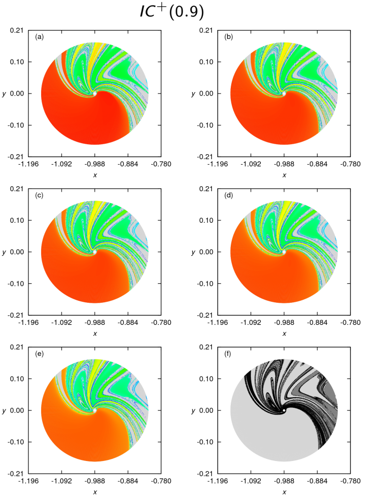
$C^{\oplus} = 3.16117289$

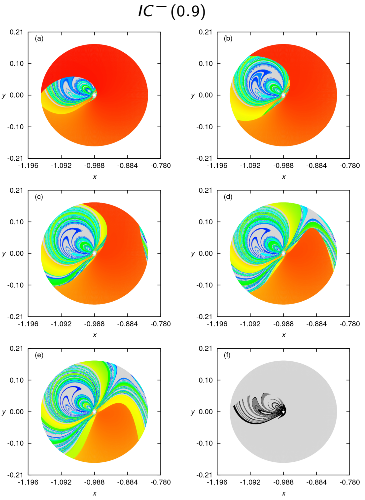
- **Hiten-like? Ballistic capture? But using manifold structure!**

W. Koon, M. Lo, J. Marsden, S. Ross (2000); W. Koon, M. Lo, J. Marsden, S. Ross (2001); P.A. Sousa Silva (2011)

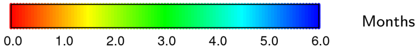
*Differential correction needed to obtain final solution!

Checks for $t < 0$: applicability

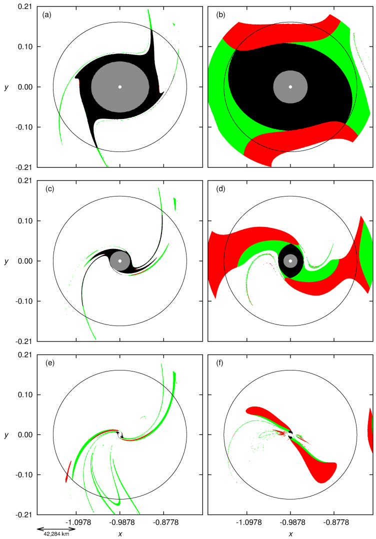




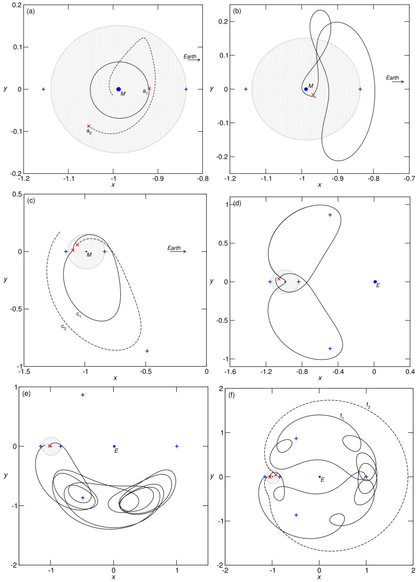
(a) Σ_1 **(b)** Σ_2 **(c)** Σ_3 **(d)** Σ_4 **(e)** Σ_5
(f) Collision (black) / None (dark gray)



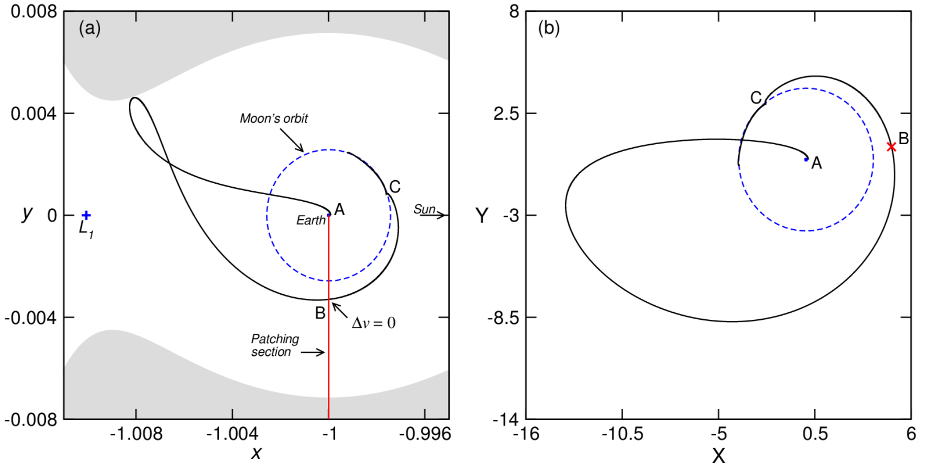
Checks for $t > 0$: stability



Red: $r_f > r_S$; Green: $r_f < r_S$; Black and Gray: $r_2(t) < r_S, \forall t \in [0, t_f]$, for $C < C_1$ and $C \geqslant C1$

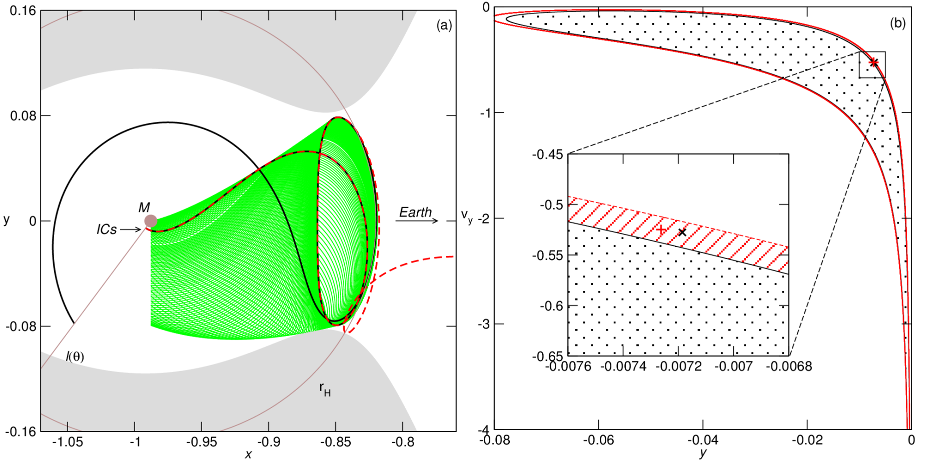


Checks for $t < 0$ + checks for $t > 0$ provide Earth-to-Moon transfers (within the Patched Three-body approach) with $\Delta v = 0$ at patching section!



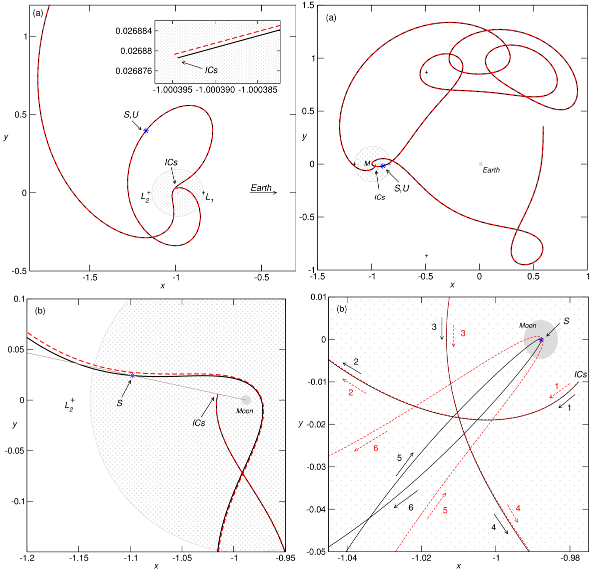
WSB corresponding to invariant manifols?

YES!

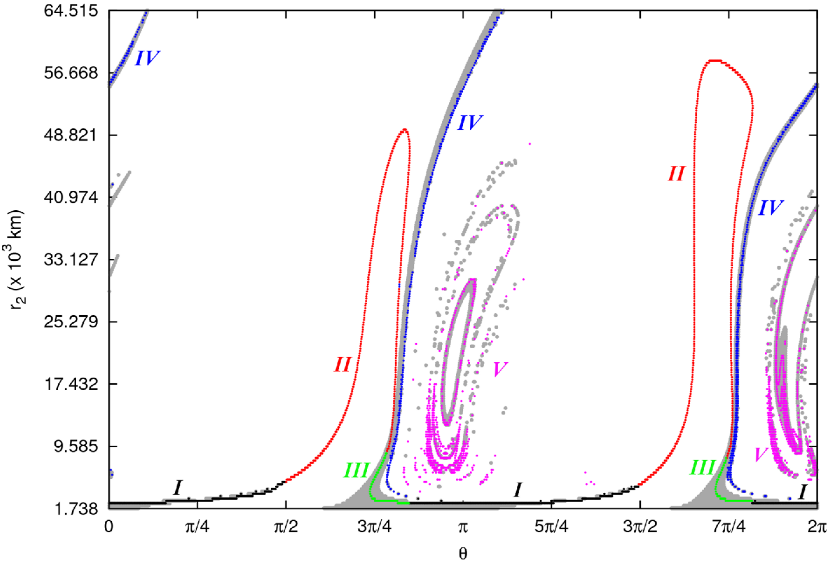


WSB corresponding to invariant manifols?

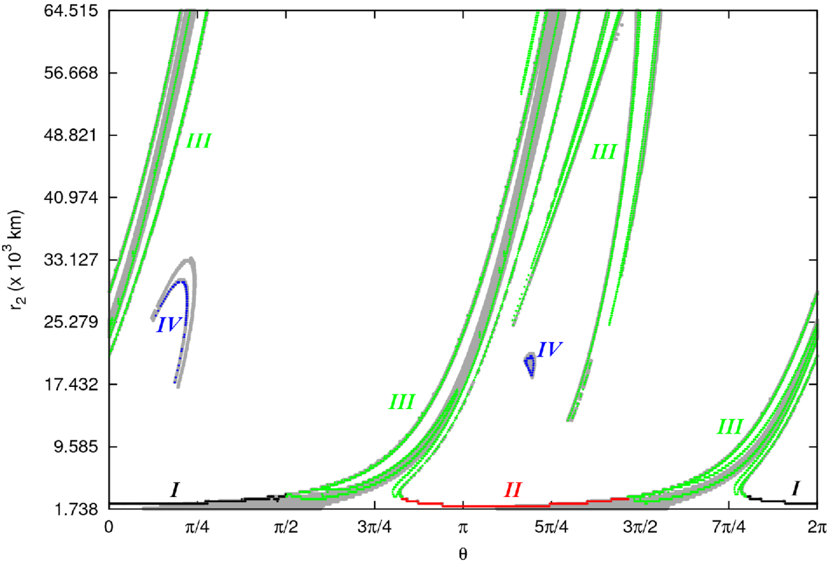
NO!



WSB corresponding to invariant manifols: up to which extent?



WSB corresponding to invariant manifols: up to which extent?



Final Remarks

- The need for new strategies for mission design lead to the use of the fundamental dynamical structure of systems of many bodies
- The WSB concept was proposed in the context of Earth-to-Moon transfers
- Our analyses showed that the current algorithmic WSB is not always related to the manifold structure of the PCR3BP
- Thus... It must be used/generalized with caution AND under some restrictions
- Our analyses provide complementary information that allow the selection of adequate stable* initial conditions for Earth-to-Moon transfer orbits in the patched-three body approach with ZERO Δv at patching!

  *(w.r.t. the stability classification associated to the WSB algorithmic definition)
- Q. Why to study WSB-like approaches if we have manifold structure?
  A. Manifold structure not always avaliable!
  A. Manifold structure too complicated to be easily obtained!

Thank you!

References

- E. Belbruno (1987) Lunar capture orbits, a method of constructing earth moon trajecto- ries and the lunar GAS mission. In Proceedings of the 19th AIAA/DGLR/JSASS International Eletric Propulsion Conference, Colorado, 1987, Article AIAA/87-1054
- E. Belbruno, J. Miller (1990) A Ballistic Lunar Capture Trajectory for the Japanese Spacecraft Hiten. Jet Propulsion Laboratory, Pasadena, 1990. Report No. 312/90.4-1731-EAB
- E. Belbruno, J. Miller (1993) Sun-perturbed Earth-to-Moon transfers with ballistic capture. Journal of Guidance, Control and Dynamics, v. 16, p. 770-775
- E. Belbruno, F. Topputo, M. Gidea (2008) Resonance transitions associated to weak capture in the restricted three-body problem. Advances in Space Research, v. 42, p. 1330-1351
- E. Belbruno (2004) Capture Dynamics and Chaotic Motions in Celestial Mechanics. Princeton University Press
- F. García, G. Gómez (2007) A note on weak stability boundary. Celestial Mechanics and Dynamical Astronomy, v. 97, p. 87-100.
- C. Conley (1968) Low energy transit orbits in the restricted three-body problem. SIAM Journal of Applied Mathematics, v. 16, p. 732-746
- R. McGehee (1969) Some homoclinic orbits for the restricted three-body problem. PhD thesis, University of Wisconsin, Madison.
- W. Koon, M. Lo, J. Marsden, S. Ross (2000) Shoot the Moon. In Proceedings of AAS/AIAA Space Flight Mechanics Meeting, Article AAS 00-166
- W. Koon, M. Lo, J. Marsden, S. Ross (2001) Low energy transfer to the Moon. Celestial Mechanics and Dynamical Astronomy 81, p. 63-73
- P.A. Sousa Silva (2011) The algorithmic WSB in Earth-to-Moon mission design: dynamical aspects and applicability. PhD thesis, Instituto Tecnológico de Aeronáutica - São José dos Campos

The Restricted Three-body Problem

Equations of motion of P_3

$$\ddot{x} - 2\dot{y} = \Omega_x, \qquad \ddot{y} + 2\dot{x} = \Omega_y,$$

with $$\Omega = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{\mu(1-\mu)}{2},$$

$$r_1^2 = (x-\mu)^2 + y^2, \text{ and } r_2^2 = (x+1-\mu)^2 + y^2.$$

The integral of motion

$$J(x, y, \dot{x}, \dot{y}) = 2\Omega(x, y) - (\dot{x}^2 + \dot{y}^2) = C, \quad C \text{ is the Jacobi constant.}$$

$$\mathcal{M}(\mu, C) = \left\{(x, y, \dot{x}, \dot{y}) \in \mathbb{R}^4 | J(x, y, \dot{x}, \dot{y}) = constant\right\}.$$

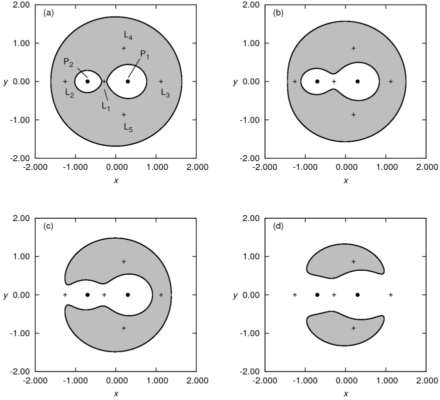
Equilibrium points

- $L_{1,2,3}$: collinear points, saddle-center.
- $L_{4,5}$: triangular points, stable if $m_1/m_2 > 24.96$.

The Restricted Three-body Problem

Hill regions, $\mathcal{H}$

- Accessible areas for each C: $\mathcal{H}(\mu, C) = \{(x, y) | \Omega(x, y) \geqslant C/2\}$
- Bounded by the *zero-velocity curves*



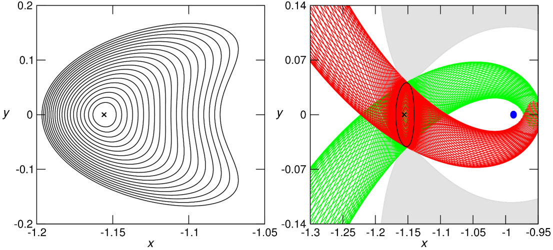
For a given μ, there are five different configurations for $\mathcal{H}$:

Case 1: $C > C_1$;
Case 2: $C_1 > C > C_2$;
Case 3: $C_2 > C > C_3$;
Case 4: $C_3 > C > C_4 = C_5$;
Case 5: $C_4 = C_5 > C$ - motion over the entire x-y plane is possible.

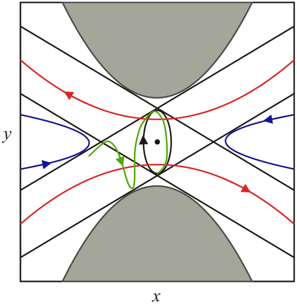
C_k, $k = 1, 2, 3, 4, 5$ denote the Jacobi constant values at L_K.

The Restricted Three-body Problem

Lyapunov Orbits



- Moser (1958) and Conley (1968,1969): existence of unstable periodic orbits around the collinear equilibria.



- Types of solution around the equilibria: **periodic**, **transit**, **asymptotic** and **non-transit**.
- Stable manifold (green): $W^s(\Gamma) = \{\mathbf{x} \in \mathbb{R}^4 : \phi(\mathbf{x}, t) \to \Gamma, t \to \infty\}$;
- Unstable manifold (red): $W^u(\Gamma) = \{\mathbf{x} \in \mathbb{R}^4 : \phi(\mathbf{x}, t) \to \Gamma, t \to -\infty\}$.

W^s and W^u are locally homeomorphic to 2D cylinders and act as separatrices of the phase space.