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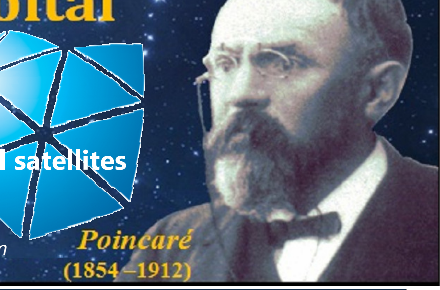

Semi-analytical study of the stability of the rotational motion of artificial satellites subject to external torques using quaternions

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Introduction

This study aims to perform the stability analysis of the rotational motion to artificial satellites using quaternions to describe the satellite attitude (orientation on the space). In the system of rotational motion equations, which is composed by the four kinematic equations of the quaternions and by the three Euler equations in terms of the rotational spin components. The torques that we have been considered are the Gravity Gradient (GGT) and The Direct Solar Radiation Pressure (DSRPT). Equilibrium points are obtained through numerical simulations using the softwares MATLAB and OCTAVE, which are then analyzed by the Routh-Hurwitz Stability Criterion.

Attitude Quaternions and Rotational Motion Equations

Attitude Quaternions [1]

$$\hat{n} = (n_1\ n_2\ n_3)^t \qquad q = (\bar{q}\ q_4)^t = (q_1\ q_2\ q_3\ q_4)^t \qquad (1)$$

$$q_1 = n_1\ sen(\tfrac{\phi}{2}) \quad q_2 = n_2\ sen(\tfrac{\phi}{2}) \quad q_3 = n_3\ sen(\tfrac{\phi}{2}) \quad q_4 = \cos(\tfrac{\phi}{2}) \qquad (2)$$

Motion Equations

Kinematic Equations [2]

$$\begin{aligned} \dot{q}_1 &= 0.5[p q_4 - q q_3 + r q_2] \\ \dot{q}_2 &= 0.5[q q_4 - r q_1 + p q_3] \\ \dot{q}_3 &= 0.5[r q_4 - p q_2 + q q_1] \\ \dot{q}_4 &= -0.5[p q_1 + q q_2 + r q_3] \end{aligned} \qquad (3)$$

Euler Equations [3]

$$\begin{aligned} I_x \dot{p} &= M_x + (I_y - I_z) q r \\ I_y \dot{q} &= M_y + (I_z - I_x) p r \\ I_z \dot{r} &= M_z + (I_x - I_y) p q \end{aligned} \qquad (4)$$

Gravity Gradient Torque

Gravity Gradient Elementar Force over an infinitesimal area element (dS) of the satellite surface, is given by [1, 2]:

$$d\vec{F} = -\mu dm \frac{\vec{r}_T}{r_T^3} \qquad (5)$$

Gravity Gradient Torque over an infinitesimal area element (dS) of the satellite surface, is given by:

$$d\vec{N}_G = \vec{r}_o \times d\vec{F} \;\Rightarrow\; \vec{N}_G = 3\frac{\mu}{r'^3}\left[a_{21}a_{31}(I_z - I_y)\hat{i}_g + a_{11}a_{31}(I_x - I_z)\hat{j}_g + a_{11}a_{21}(I_y - I_x)\hat{k}_g\right] \qquad (6)$$

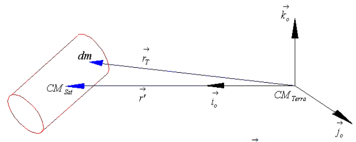

Fig. 1: Relations between the mass elements of the satellite and the Earth, being $\vec{r}'$ the satellite center of mass position vector related to the Earth center of ma $\vec{i}_o$ and the unit vector in its direction.

Solar Radiation Torque (SRT)

Solar Radiation Pressure Elementar Force over an infinitesimal area element (dS) of the satellite surface [4]:

$$d\vec{F} = -\frac{\bar{K}}{R^2}\left\{\left[\frac{2\gamma}{3}(1-\beta)\cos\theta_p + 4\beta\gamma\cos^2\theta_p\right]\hat{n} + \left[(1-\beta\gamma)\cos\theta_p\right]\hat{u}\right\}dS \qquad (7)$$

Direct Solar Radiation Pressure Torque over an infinitesimal area element (dS) of the satellite surface:

$$d\vec{N} = \vec{r} \times d\vec{F} \qquad (8)$$

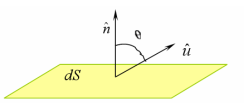

Fig. 2: Solar radiation incidence over *dS* [5]

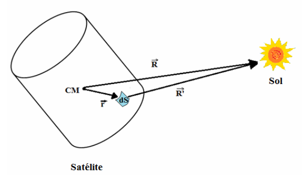

Fig. 3: CM and *dS* positions related to the Sun [6]

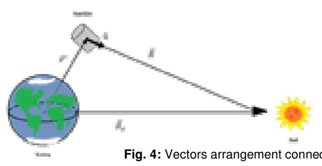

Fig. 4: Vectors arrangement connecting the satellite, the Earth and the Sun [6]

$$\vec{N} = N_x\hat{e}_x + N_y\hat{e}_y + N_z\hat{e}_z \qquad (9)$$

$$N_x = -\frac{\bar{K}}{R^4}(\beta_1\gamma_1 - \beta_2\gamma_2)\frac{h}{2}\pi\sigma^2 G_1 \qquad N_y = -\frac{\bar{K}}{R^4}(\beta_1\gamma_1 - \beta_2\gamma_2)\frac{h}{2}\pi\sigma^2 G_2 \qquad N_z = 0 \qquad (10)$$

Algorithm for Stability Analysis

The Routh-Hurwitz Criterion allows to investigate the absolute stability of a system of equations using the coefficients of the characteristic equation associated with the linearized system, without the need to determine the roots of the characteristic equation. [7, 8].

$$a_n \lambda^n + a_{n-1}\lambda^{n-1} + ... + a_2\lambda^2 + a_1\lambda + a_0 = 0 \qquad (11)$$

Routh Table, general case:

k = i - 1 \ j		1	2	3 ...	j
0	λ^n :	a_n	a_{n-2}	a_{n-4}	a_{n+2-2j}
1	λ^{n-1} :	a_{n-1}	a_{n-3}	a_{n-5}	a_{n+1-2j}
2	λ^{n-2} :	b_{n-2}	b_{n-4}	b_{n-6}	b_{n-2j}
3	λ^{n-3} :	c_{n-3}	c_{n-5}	c_{n-7}	c_{n-1-2j}
k	λ^{n-k} :	Z_{n-k}	Z_{n-k-2}	Z_{n-k-4}	$Z_{n-k+2-2j}$
n - 4	λ^4 :	α_4	α_2	α_0	
n - 3	λ^3 :	β_3	β_1		
n - 2	λ^2 :	γ_2	γ_0		
n - 1	λ^1 :	δ_1			
n	λ^0 :	ε_0			

Routh Table, case n = 7:

λ^7	a_7	a_5	a_3	a_1
λ^6	a_6	a_4	a_2	a_0
λ^5	b_5	b_3	b_1	
λ^4	c_4	c_2	c_0	
λ^3	d_3	d_1		
λ^2	e_2	e_0		
λ^1	f_1			
λ^0	g_0			

Results obtained by numerical simulations

Table 1: Order of Magnitude of the eigenvalues with real part less then 10^{-5}

	Order of magnitude of the eigenvalues real part (λ)							
Satellite	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}	10^{-10}	10^{-14}	TOTAL
MP	9	2	2	0	1	0	0	14
PP-1	36	0	10	2	4	2	2	56
PP-2	39	1	3	1	3	1	1	49
TOTAL	85	3	15	3	7	3	2	119

Table 2: Satellite PP-2 equilibrium point

Equilíbrium point components	Values obtained in the simulations
p (rad/s)	$4,239\times10^{-13}$
q (rad/s)	$1,578\times10^{-13}$
r (rad/s)	$-9,198\times10^{-13}$
q_1	$9,001\times10^{-2}$
q_2	$9,001\times10^{-2}$
q_3	$9,001\times10^{-1}$
q_4	$6,079\times10^{-12}$

Table 3: 1st Column of the Routh Table for a PP-2 equibrium point

Coefficients of the first column	Values obtained in the simulations
a_0	1,000
a_1	$7,100\times10^{-20}$
b_1	-1,208
c_1	$8,577\times10^{-20}$
d_1	$-1,995\times10^{-14}$
e_1	$1,340\times10^{-26}$
f_1	$1,481\times10^{-22}$
g_1	$1,455\times10^{-36}$

Table 4: Eigenvalues associated to the PP-2 instable point

	Eigenvalues for the instable point	
	Real Part	Imaginary Part
1	$8,697\times10^{-14}$	$6,528\times10^{-4}$
2	$8,697\times10^{-14}$	$-6,528\times10^{-4}$
3	$6,600\times10^{-14}$	$3,108\times10^{-4}$
4	$6,600\times10^{-14}$	$-3,108\times10^{-4}$
5	$-1,482\times10^{-13}$	$6,101\times10^{-5}$
6	$-1,482\times10^{-13}$	$-6,101\times10^{-5}$
7	$-9,489\times10^{-15}$	0

Conclusions

- The main significance of this work is due to the use of quaternions in determining the SRT components and in a preliminary analysis of stability. Previous works used Andoyers variables and Euler angles.
- The approach taken can be improved when in the stability analysis are included other environmental torques as a mean to obtain a better representation of the real behavior of the satellite, and also to introduce the Shadow Function in the DSRPT model.
- Other methods of stability analysis can also be used as the Method, the Liapunov Direct Method and the Kovalev SavchenkoStability Theorem [9,10].
- The latter method requires the determination of the equations of motion in its canonical form, and it is therefore also a challenge to obtain the canonical form for these equations of the quaternions.
- The approach taken here is applyed to satellites with cylindrical form in the iluminated phase of the trajectory. It can be useful for analyzing the stability of future missions of the

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