Symplectic Maps for Tokamaks

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XVI CBDO Serra Negra, 2012

I- Introduction

References

Divertor

T. Kroetz, M. Roberto, I. L. Caldas, R. L. Viana, P. J. Morrison
Nuclear Fusion, 2010; Plasma Physics and Controlled Fusion, 2012

Reviews

J. S. E Portela, I. L. Caldas, R. L. Viana
European Journal of Physics, Special Topics, 2008

I. L. Caldas et al.
Plasma Physics and Controlled Fusion, 2012

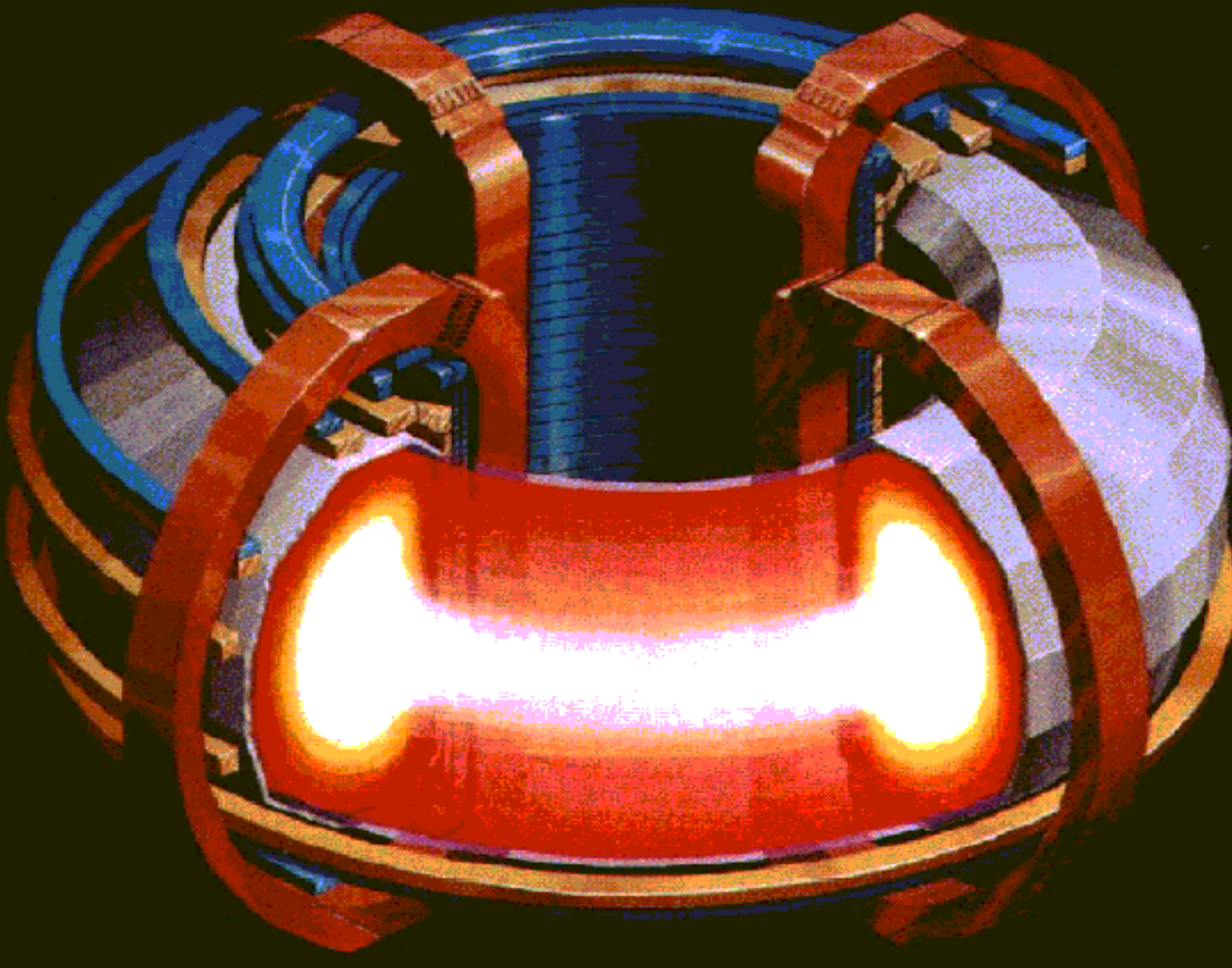
Objective

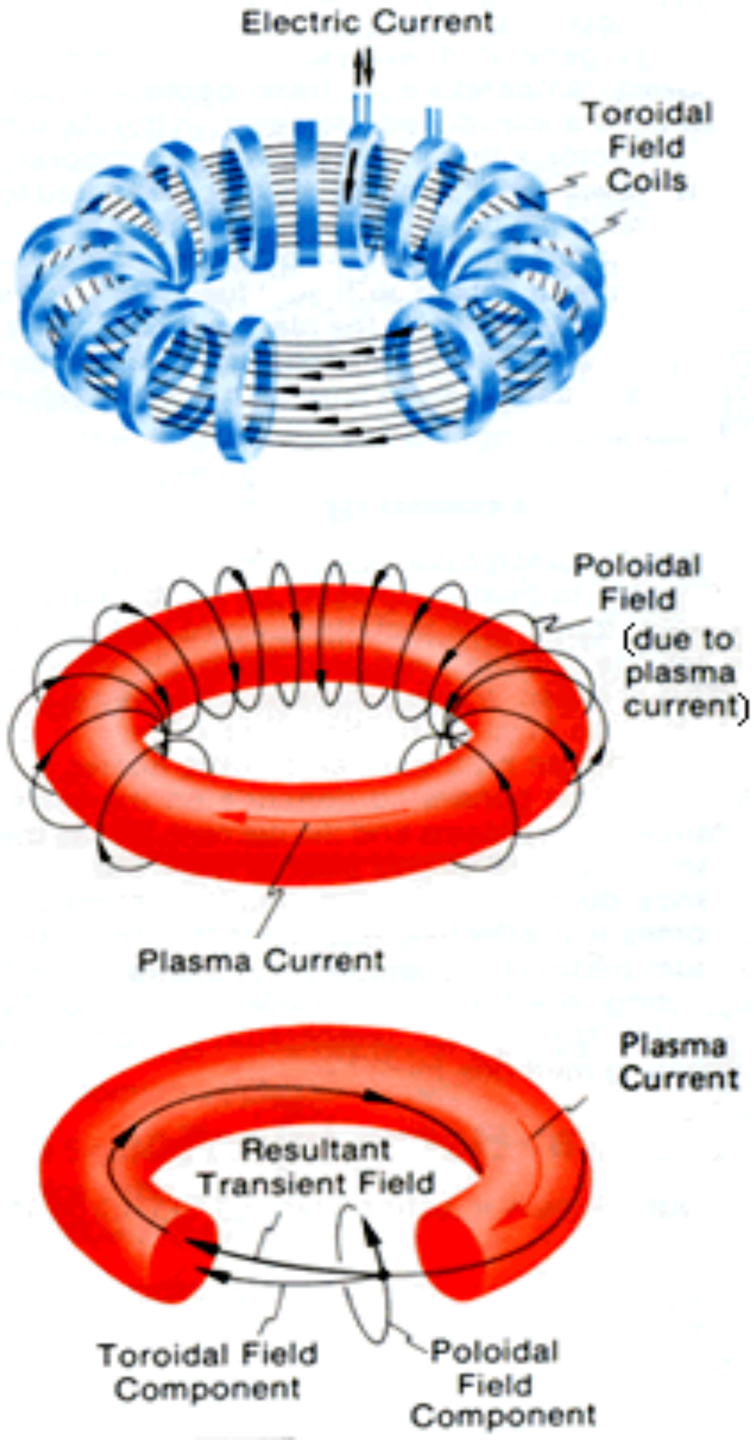
- Study magnetic field lines near the separatrix of a tokamak single-null poloidal divertor.

Methodology

- Symplectic map
- integrable map + resonant perturbation.

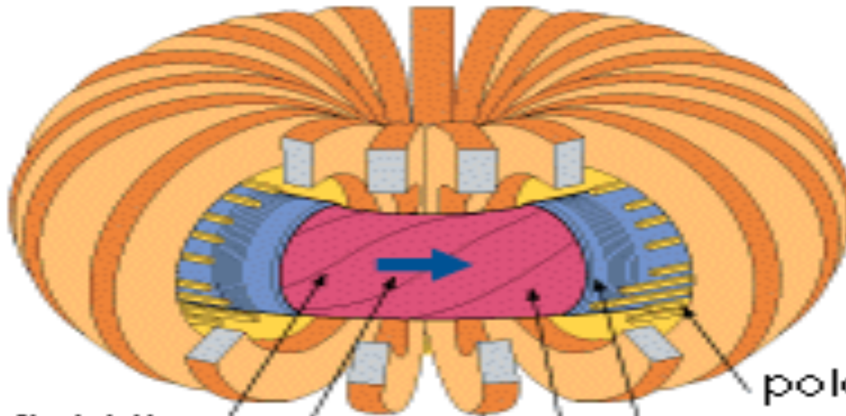
I- Magnetic Surfaces in Tokamaks




Electric Current
Toroidal
Field
Coils
Poloidal
Field
(due to
plasma
current)
Plasma Current
Plasma
Current
Resultant
Transient Field
Toroidal Field
Component
Poloidal
Field
Component

Campo Magnético de Equilibrio no Tokamak

Linhas descrevem ângulos toroidal (ϕ) e poloidal (θ), em superfícies magnéticas toroidais.



Equação da linha

$$\vec{B}_0 \times d\vec{l} = 0 \quad \rightarrow$$

Campo integrável

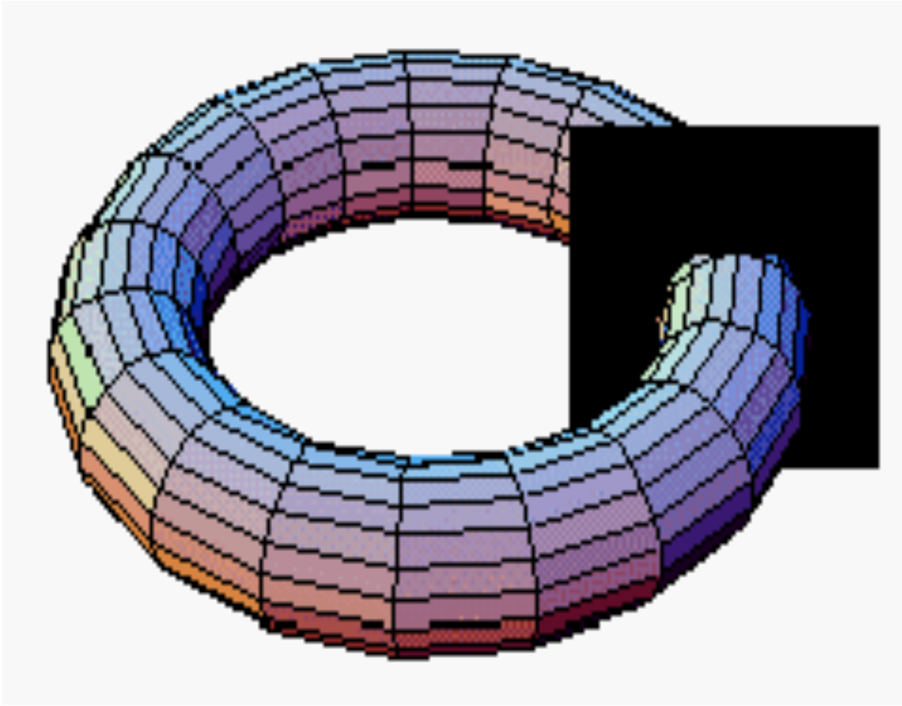
$$\dot{\vartheta} = \frac{\partial H_0(J)}{\partial J}, \quad \dot{J} = -\frac{\partial H_0(J)}{\partial \vartheta}$$

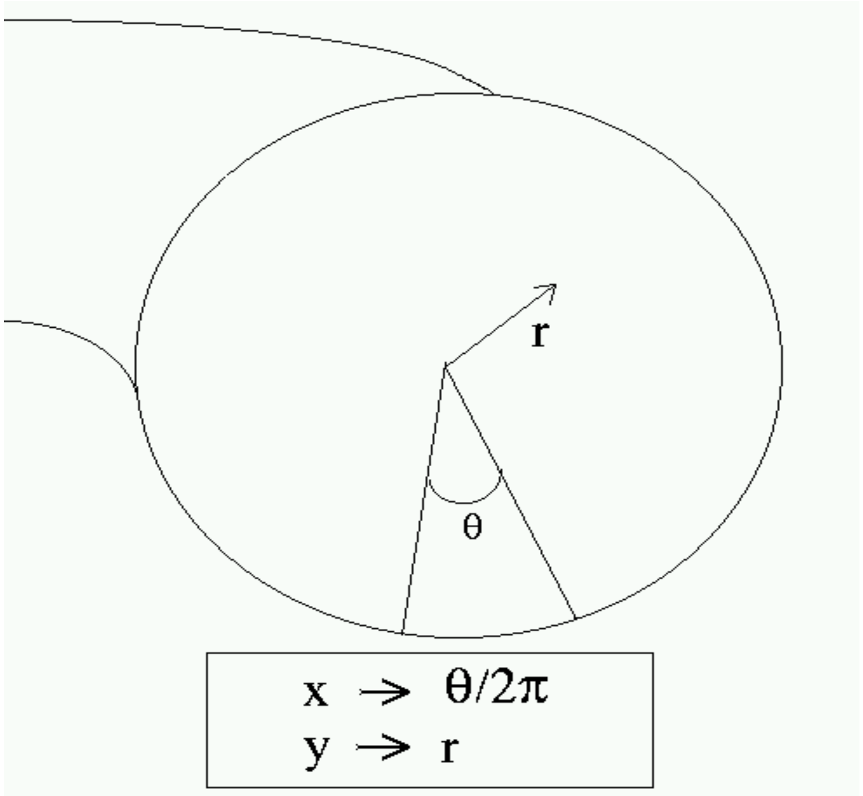
t (tempo canonico) ≡ ϕ (ângulo toroidal)

Frequência $\omega(J) = \frac{\partial H_0}{\partial J} = 1/q$

Plano para o mapa de Poincaré

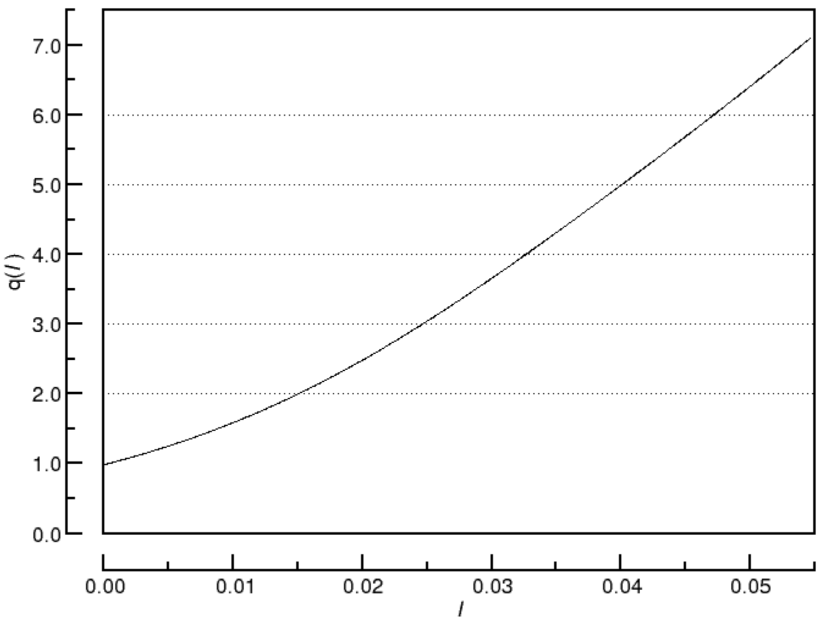
Ângulos φ (toroidal) e θ (poloidal)
m φ - n θ =0 equação da linha magnética na superfície racional com q = dθ/dφ = m/n.

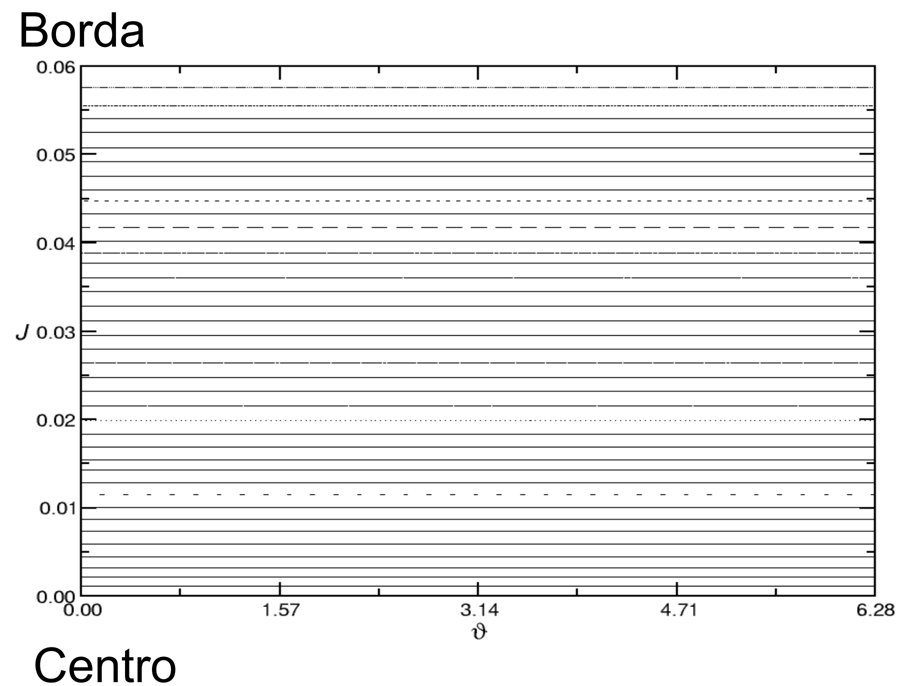




Mapa de Poincaré não perturbado

Superfície racional em q = m/1 (m ilhas a serem criadas pela perturbação ressonante com m/1).

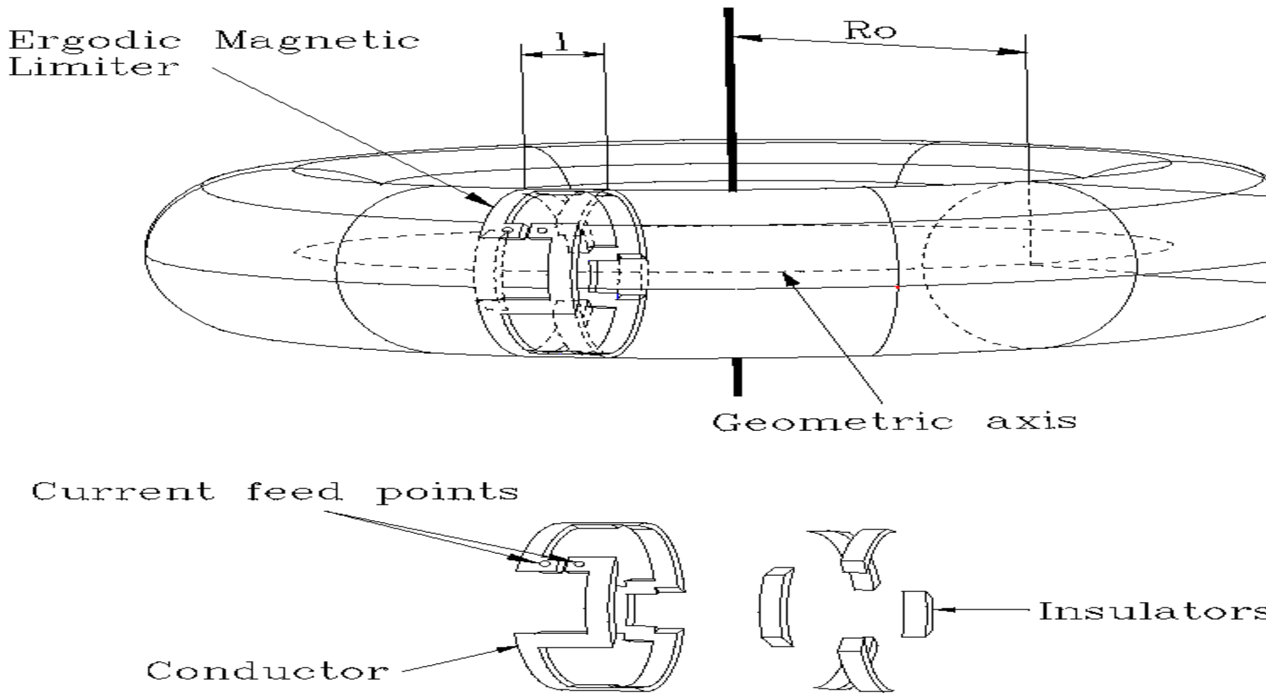




Perfil radial de $q = 1/\omega$ para equilíbrio com γ=3,25.

Ação (J) x ângulo (θ).

Limitador Caótico
Perturbações ressonantes em superfícies magnéticas



Perturbação ressonante m/n.

Parâmetro de controle: corrente no limitador I_h .

Caos Lagrangiano

$\text{Simetria} \Rightarrow \text{sistema integrável}$

$H = H_0(I) \Rightarrow \; I = I_0 \, , \vartheta = \omega\, t + \vartheta_0$

(I, ϑ) ângulo/ação de H_0

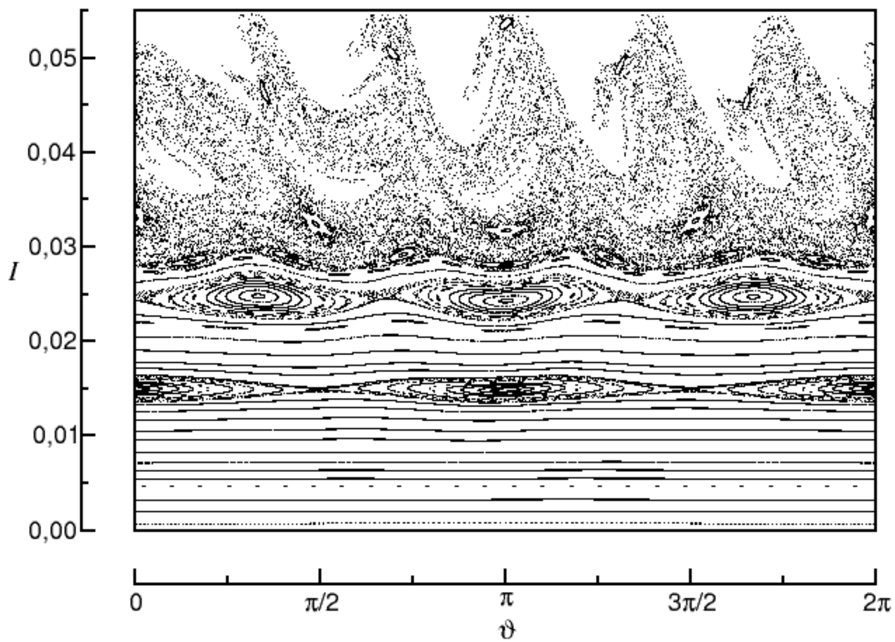
$\text{Perturbações helicoidais } (\varepsilon \neq 0) \Rightarrow \text{quebra de simetria}$

$H = H_0(I) + \varepsilon H_1(I, \vartheta)$

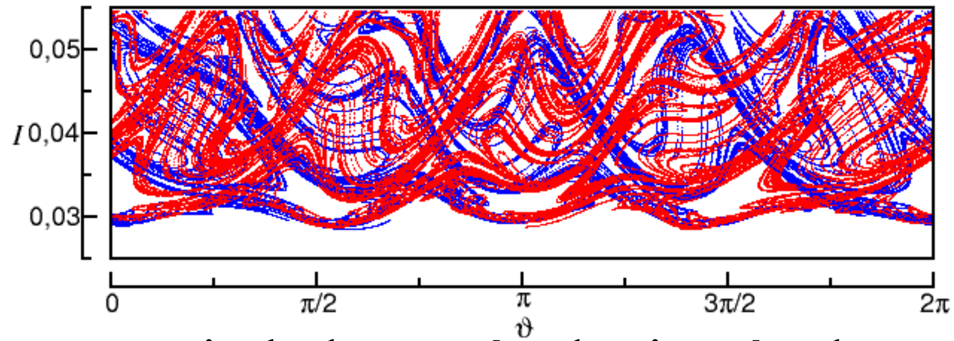
$\varepsilon << 1 \Leftrightarrow \text{sistema quase-integrável}$

Escape das Linhas Caóticas

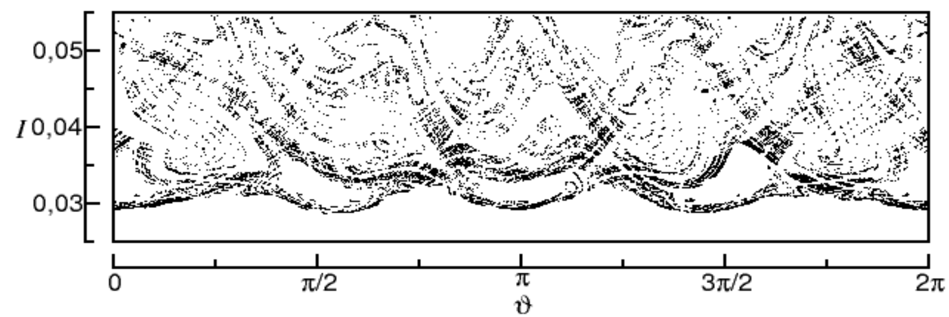
Perturbação 5/1 com $I_h/I_p = 5,5x10\text{-}2$
Equiíbrio com $\gamma = 3,25$ $\beta = 0$



Mapa de Poincaré



Variedades estável e instável



Sela caótica

III - Tokamak with Divertor

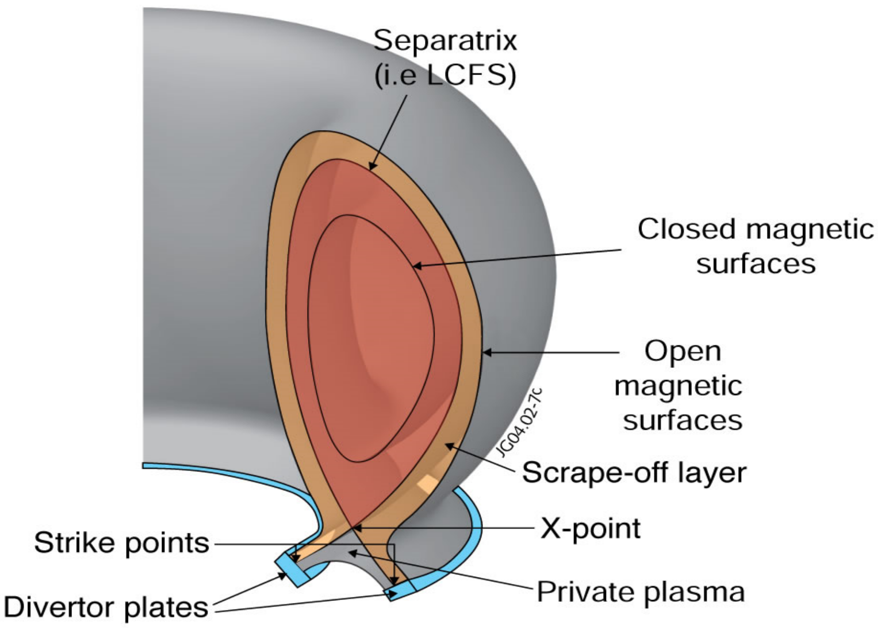
Divertor

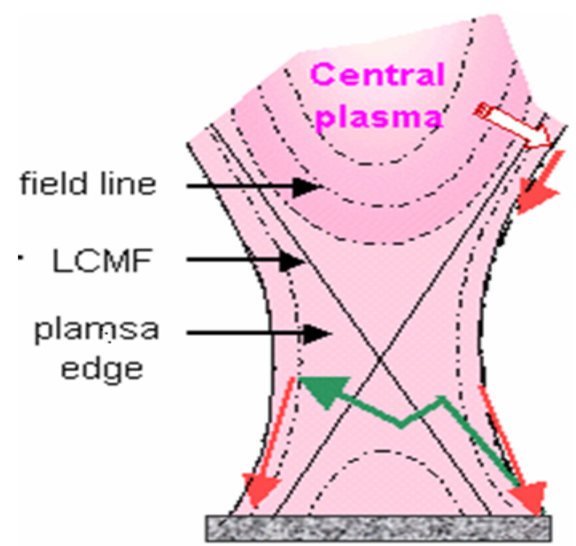
- An essential component of tokamak design.

- A metallic curved-shaped plate placed outside the plasma boundary to capture or divert particles escaping from the plasma.

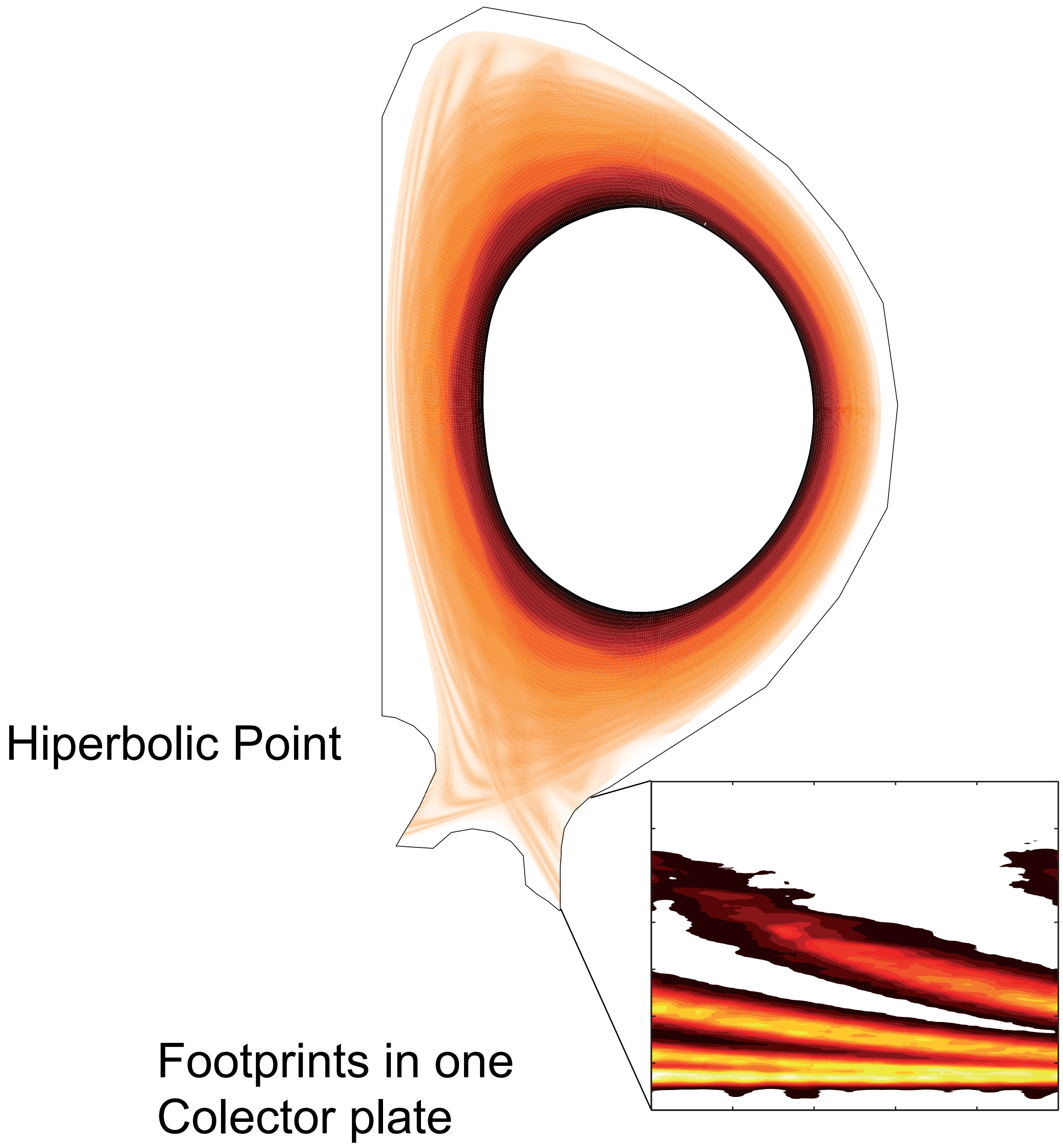
MOTIVATION:

- A magnetic frontier separates the plasma from the wall : the separatrix.
- External conductors create a point of null poloidal fields: the X point.
- Field lines of open magnetic surfaces guide the escaping particles to the collector plate, where the recombination will occur.





Tokamak with Divertor



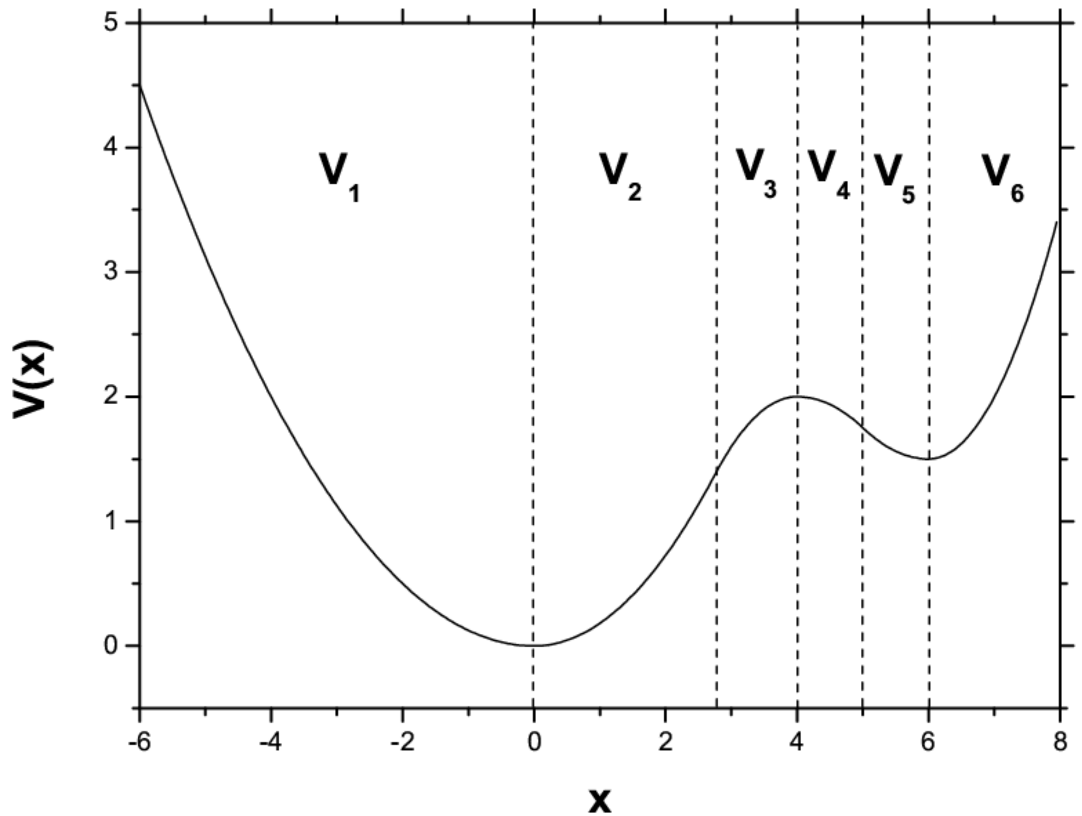
IV- Divertor Map Derivation

Flux Function for the Divertor Map

- Choose a potential V(x) that produces a topology with an X-point, with a flux function (Hamiltonian) ψ given by

$$\Psi = \frac{y^2}{2} + V(x)$$

One parabolic dependence for each V_i



The Divertor Map

• The “Trajectory Integration Method”: Next step: solve Hamilton’s equations to get x and y in terms of their initial conditions (x0,y0) and time t.

$$\psi = \frac{y^2}{2} + V(x)$$

$$\begin{cases} \dfrac{dx}{dt} = \dfrac{\partial \psi}{\partial y} \\ \dfrac{dy}{dt} = -\dfrac{\partial \psi}{\partial x} \end{cases} \rightarrow \begin{cases} x(x_0, y_0, t) \\ y(x_0, y_0, t) \end{cases}$$

• The map is obtained by the transformation: The continuous equations are transformed in a discrete map, where the continuous time parameter t is turned into a discrete time step Δ:

$$x(x_0, y_0, t) \rightarrow x_{n+1}(x_n, y_n, \Delta) \qquad y(x_0, y_0, t) \rightarrow y_{n+1}(x_n, y_n, \Delta)$$

• The Δ parameter is related to the frequency of the surface.

Divertor Symplectic Map

- For each line on the map:

  ψ is given by $\psi = \psi(\mathrm{x}_0, \mathrm{y}_0)$

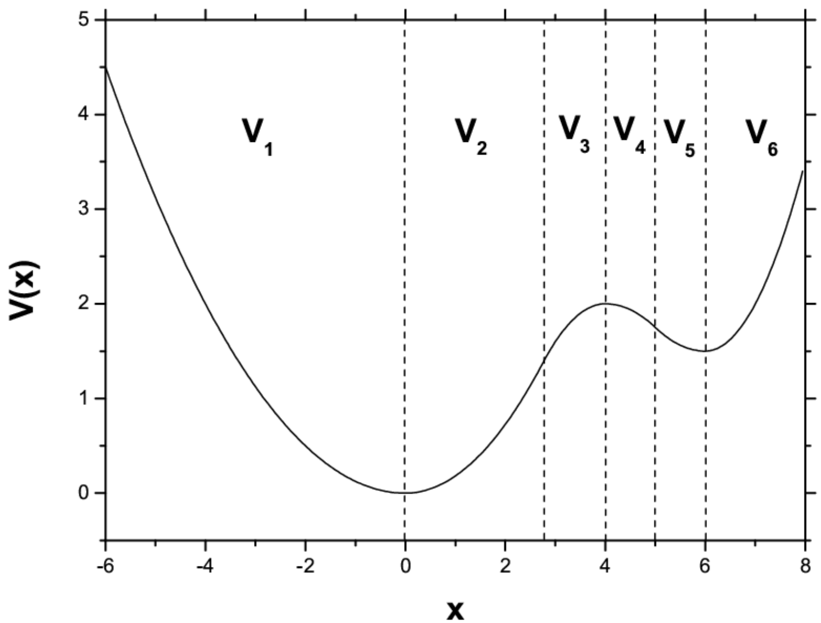
  This ψ determines the q (ψ) value and

  $\Delta = 2\pi/q(\psi)$

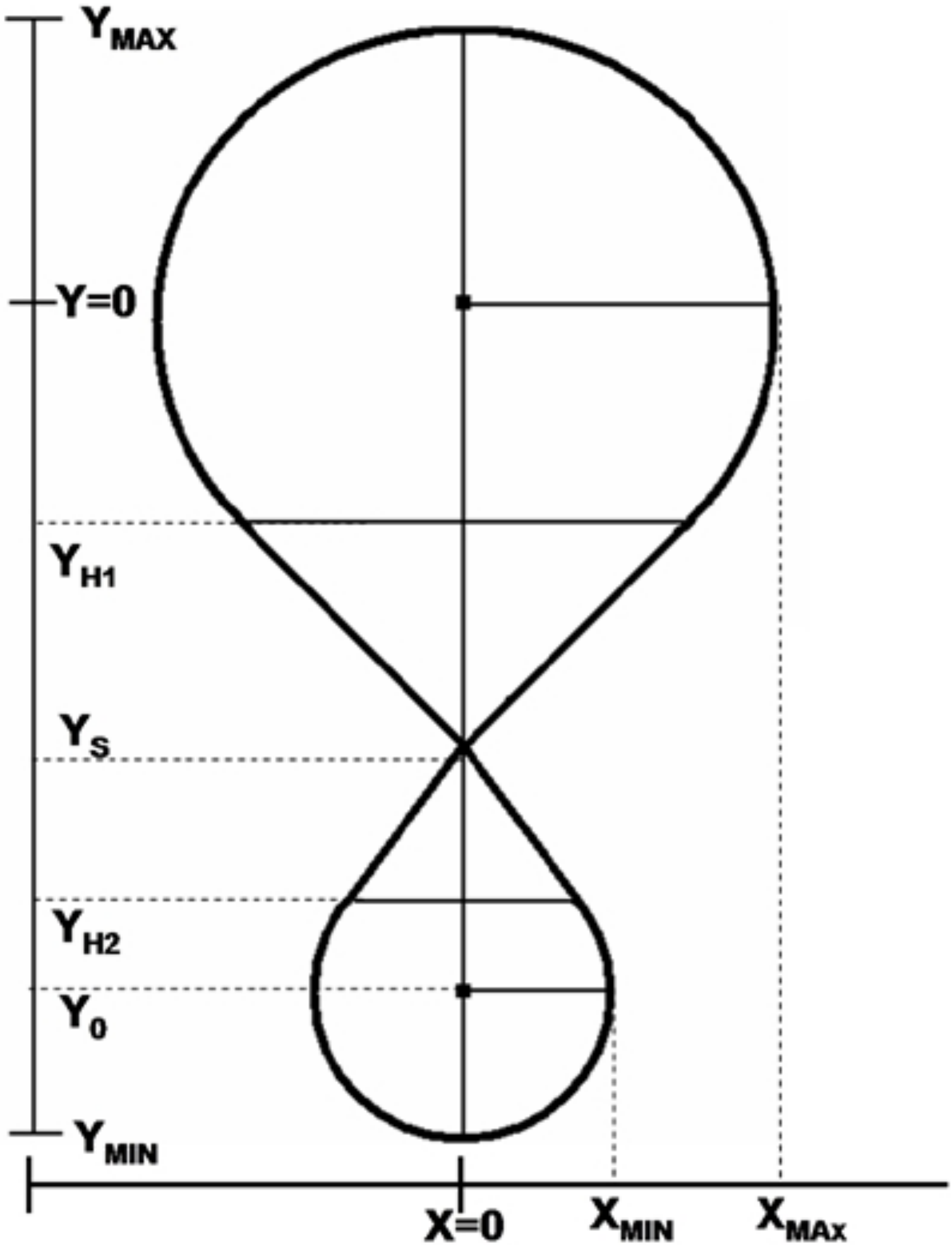
$$M_\Delta(x_n, y_n) = (x_{n+1}, y_{n+1})$$

The divertor map corresponds to the Poincaré section of magnetic surfaces at φ=0.

Separatrix geometrical parameters related to potential parameetrs



$$\psi = \frac{y^2}{2} + V(x)$$



Dependence $\Delta(\psi)$ obtained from magnetic surfaces

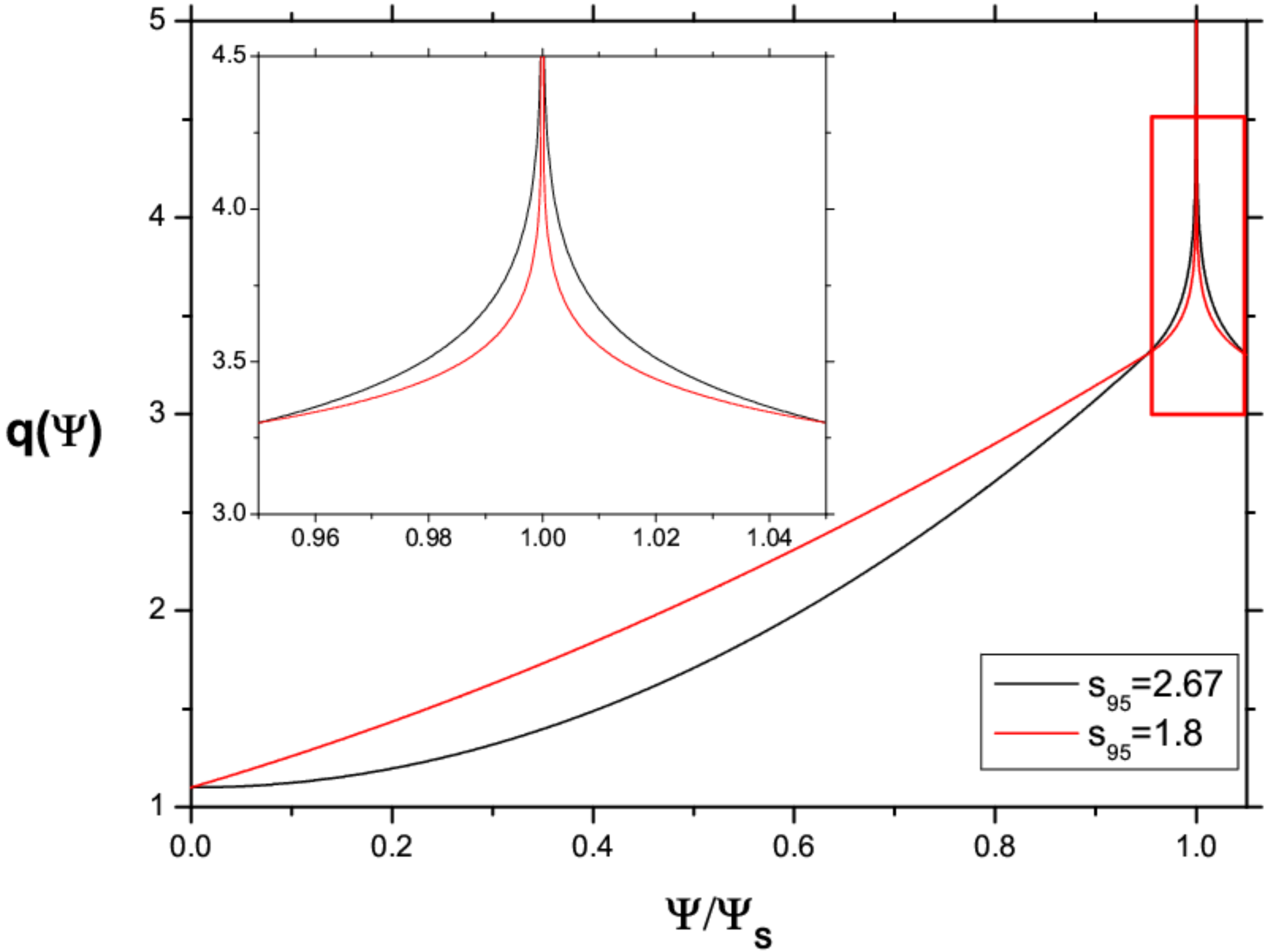
- We need to find Δ as a function of ψ, to have a map with Jacobian=1.

- The Δ parameter is related to $q(\psi)$, the frequency inverse.

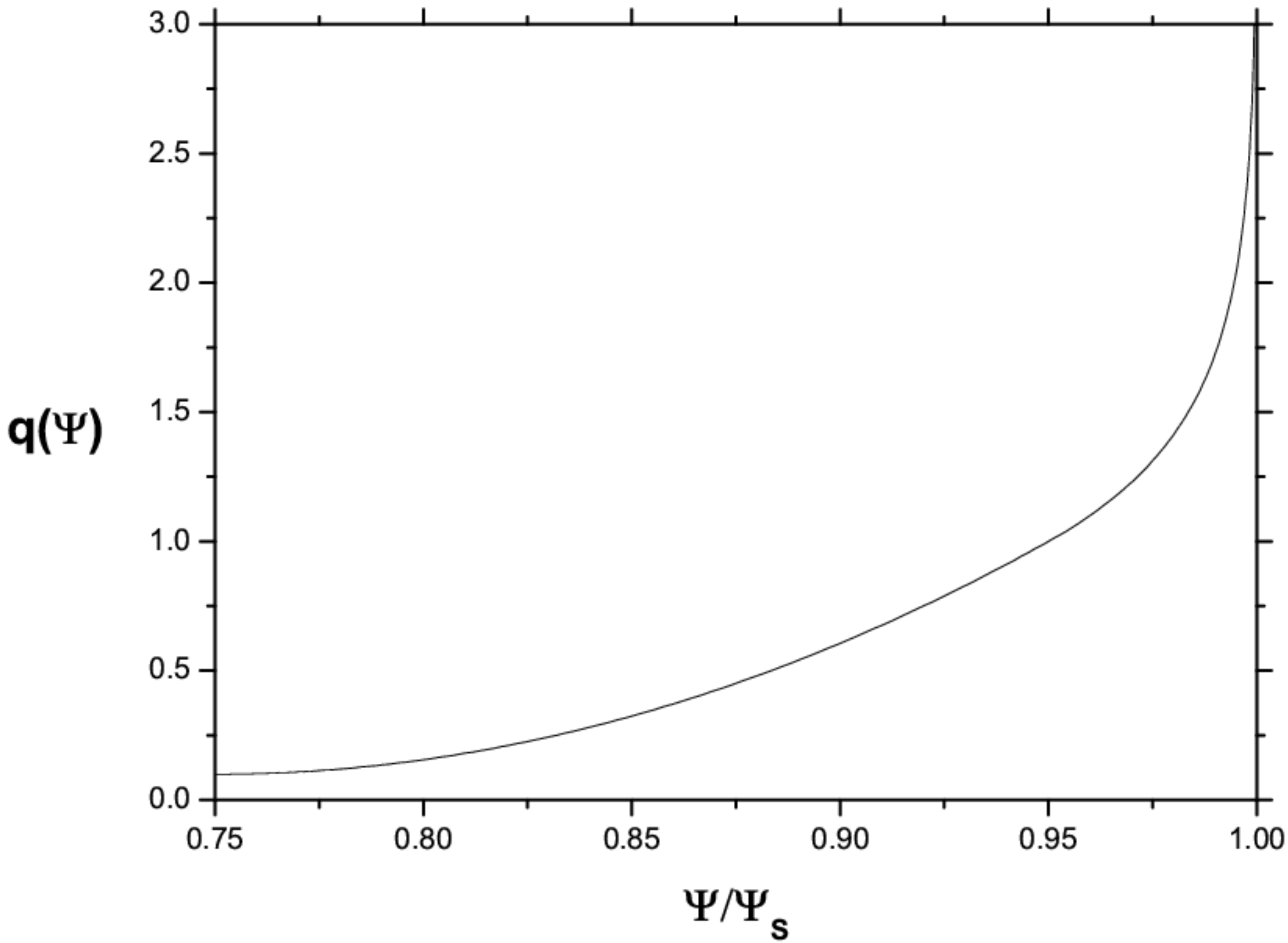
Safety Factor Profile

- Typical plasma with divertor.
- Compare chaotic divertor region for different shear and modes.

We choose $q(0)=1.1$; $q_{95}=3.3$



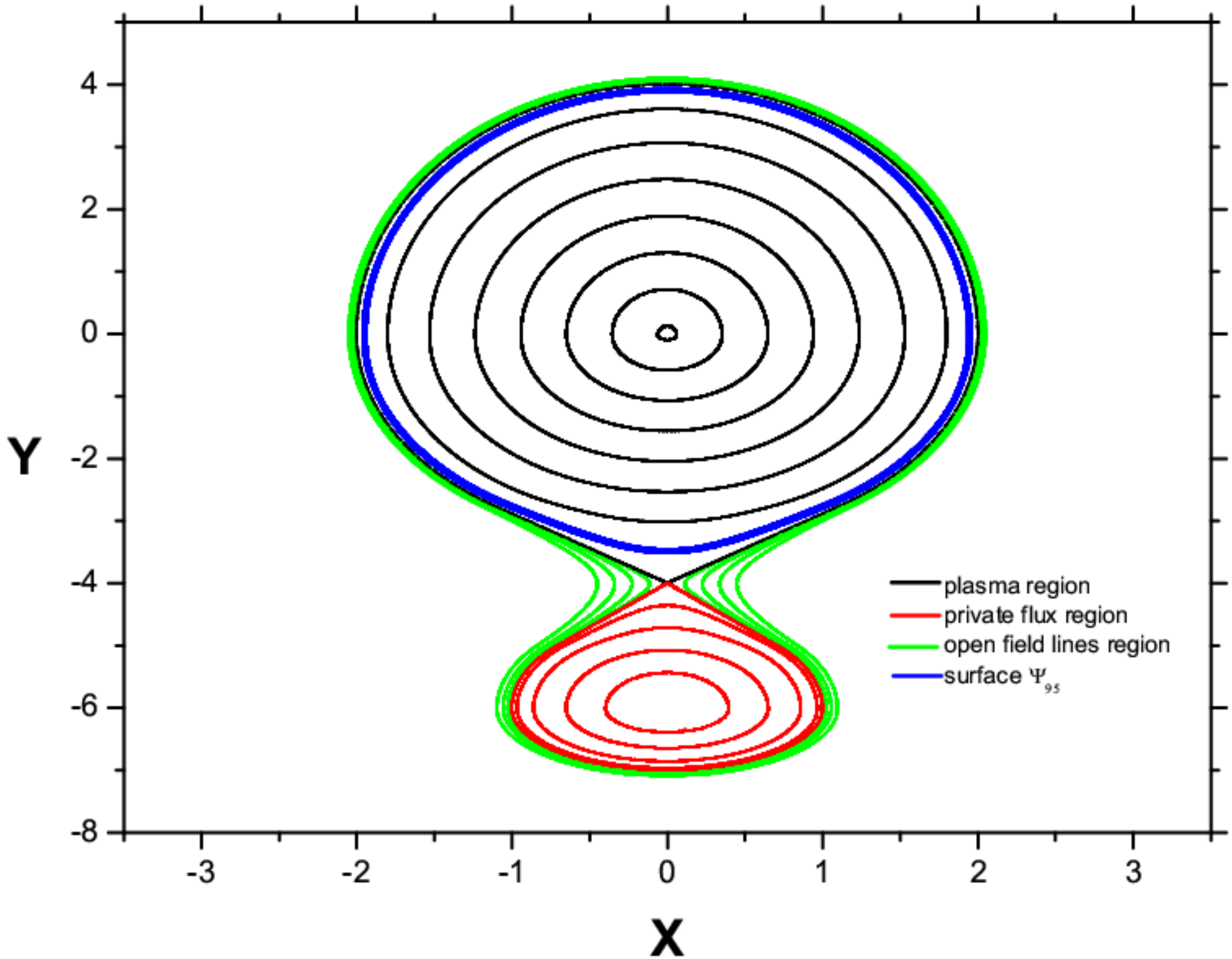
Safety factor profile plasma region ($\Psi / \Psi_S<1$) and for region external to separatriz ($\Psi / \Psi_S>1$).



Safety factor profile for divertor region. It diverges for the x_s point.

Numerical example of Divertor Map for vertical divertor

To give an exemple similar do ITER project we apply our maping to a vertical divertor

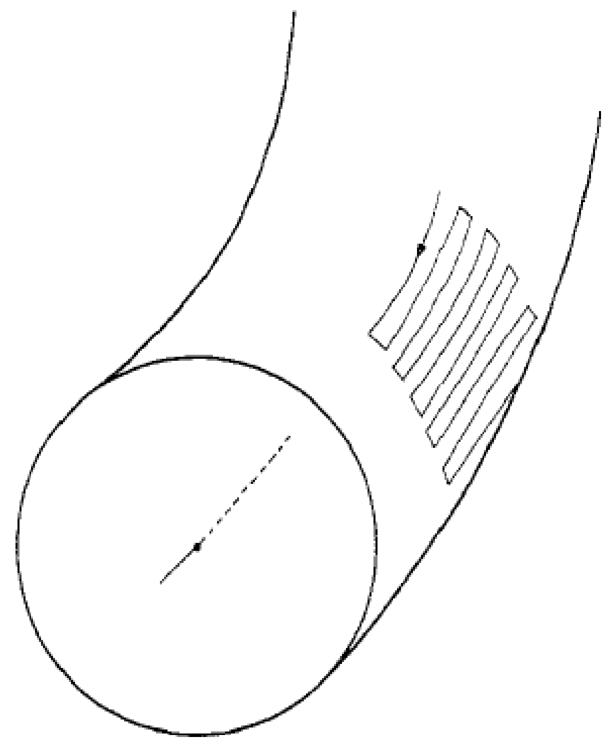


V- Perturbing Map (Martin Taylor)

Ergodic Magnetic Limiter

- The resonant perturbation was introduced by an ergodic magnetic limiter.
- For each toroidal turn Martin Taylor map was applied in φ=0 surface.
- C is the control parameter depending on perturbing current and equilibrium shear.

Cylindrical geometry



Tokamak magnetic field lines described by simple maps.
Portela, Caldas, Viana *Eur. Phys. J.: Special Topics 165 (2008)*

Map Composition

- For the next figure we consider the map convolution

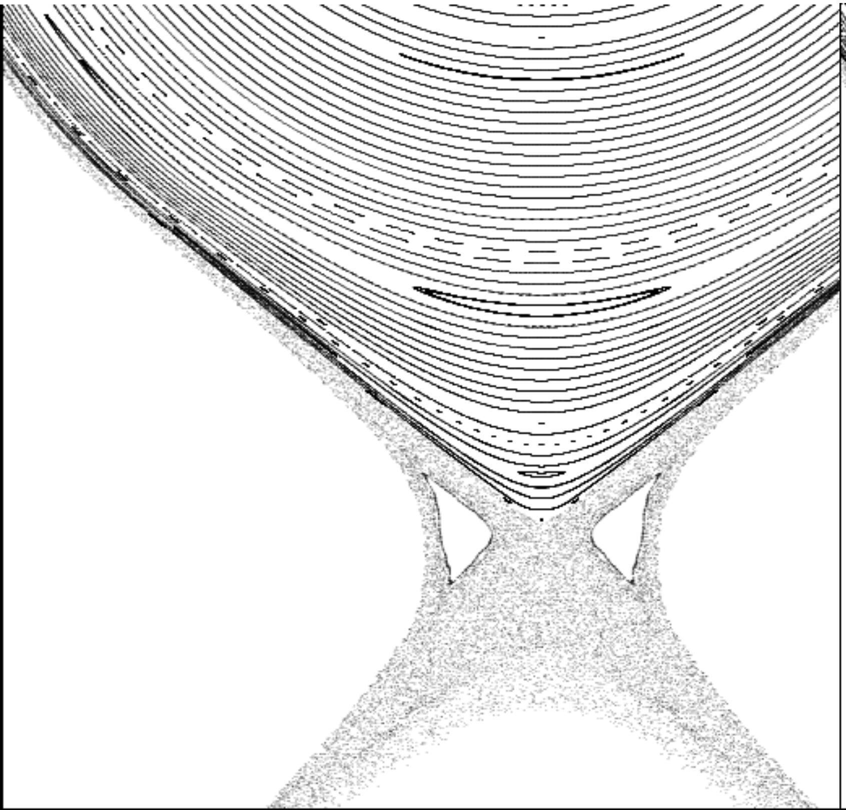
$$M = M_{divertor} \cdot M_{perturbing}$$

Perturbed Map with parameter $C=1x10^{-2}$ and m=3

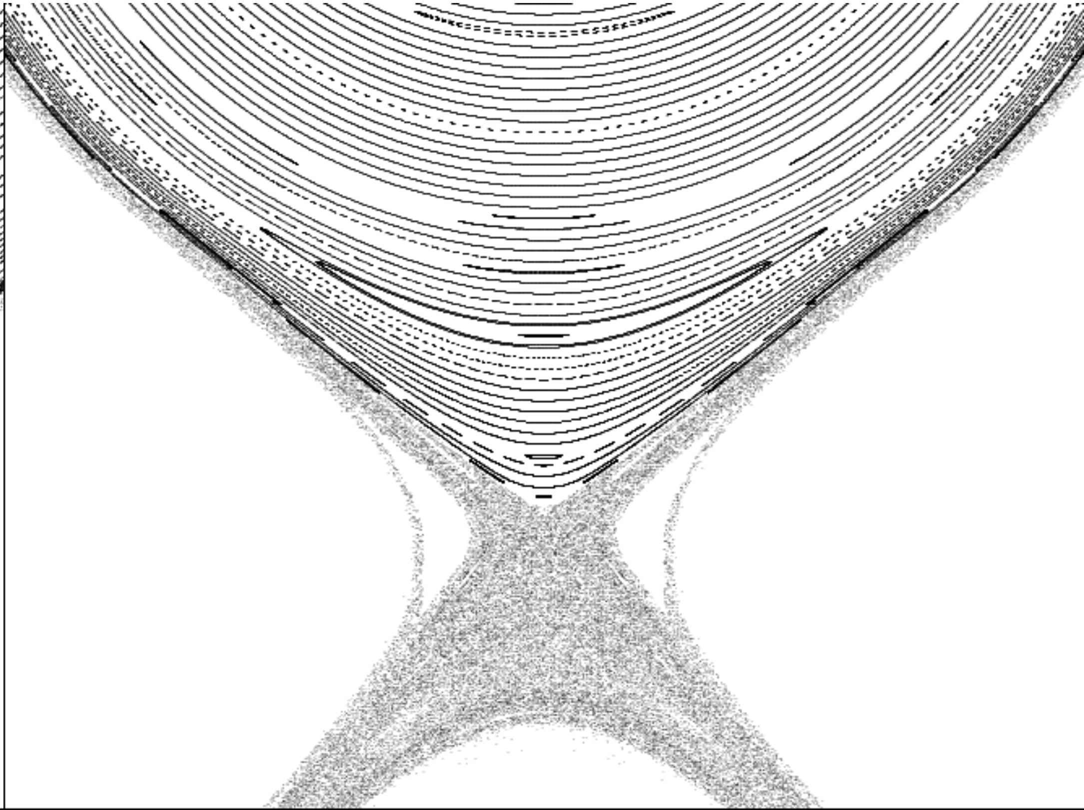
(O eixo horizontal vai de -1.5 a 1.5, e o vertical de -5 a -2)

Chaos increases with shear

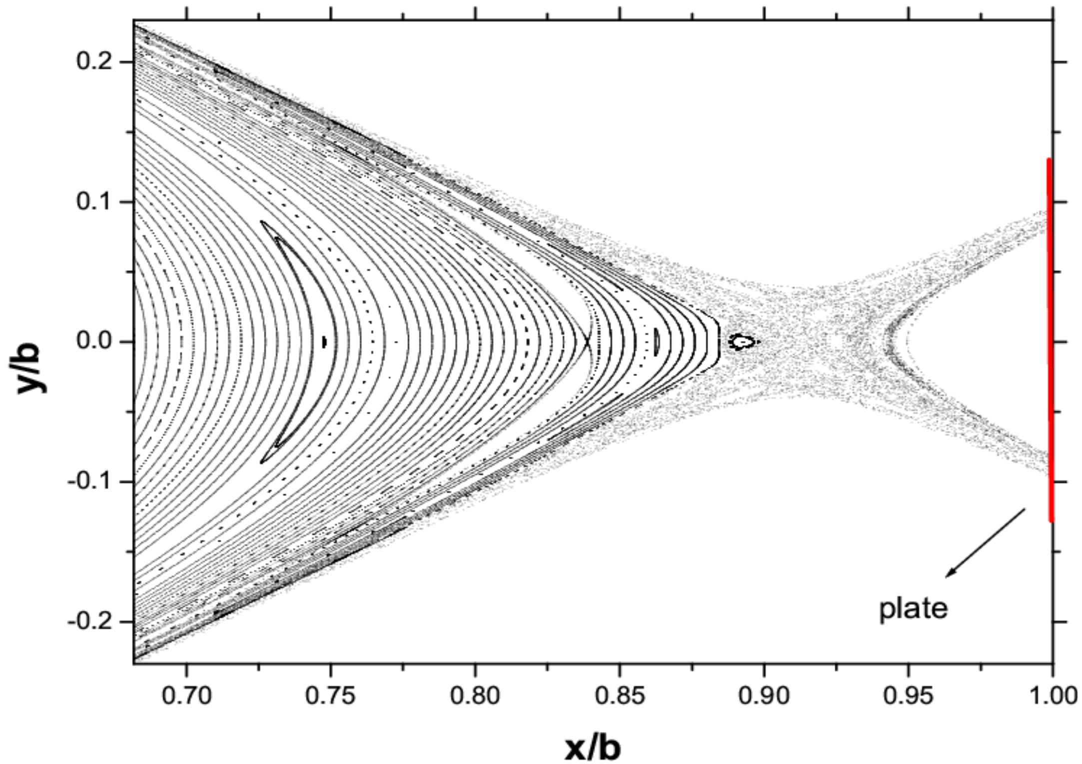
S_{95}=1.80



S_{95}=2.67



Maps for perturbation mode m=4



I_h/I_p=0.20%

Perturbed (sympletic) map $(x^*, y^*) = M_{\Delta_n}(x_n, y_n)$

$$(x_{n+1}, y_{n+1}) = P(x^*, y^*)$$

•Particle and heat deposition in the divertor plates is not uniform.

•Regions of high deposition are potentially dangerous places of intense localized loading.

•These high deposition layers possess a fractal-like structure, called magnetic footprint, which have also been experimentally investigated [DIII-D, Textor].

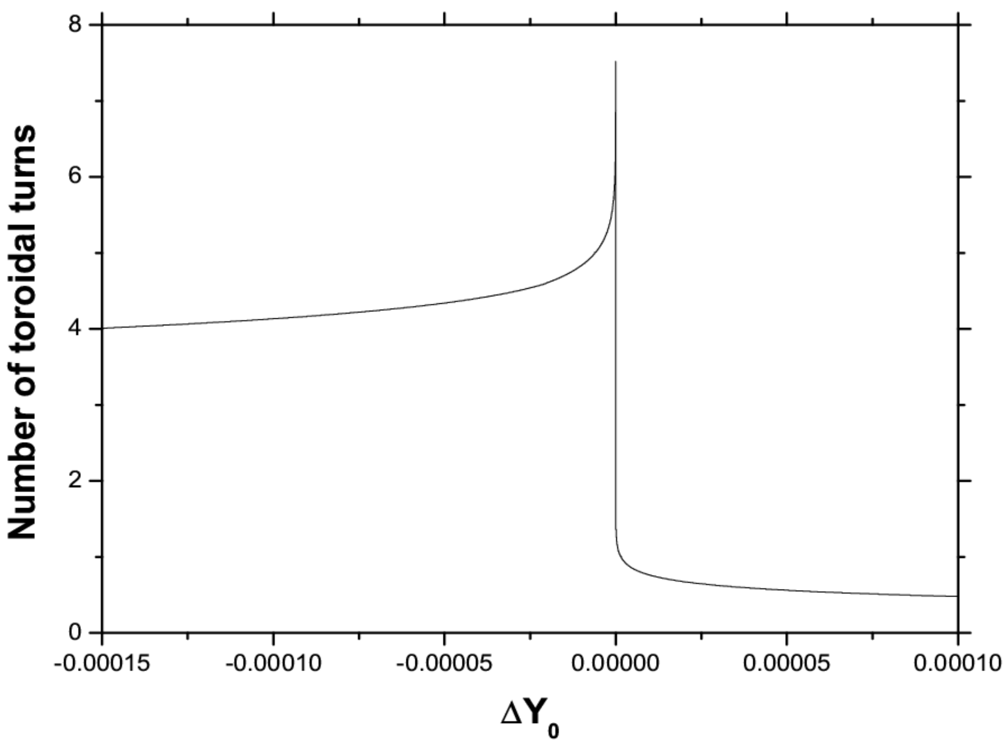
- Magnetic footprints have been explained by the underlying mathematical structure of the outer chaotic region.

- The latter is not uniform, as would be expected at first, and it possesses escape channels due to the complicated structure of invariant manifolds attached to the unstable orbits embedded in the chaotic region.

Connection Lengths

Number of toroidal turns for a field line, originating from a given initial condition, to reach the divertor plate.

$I_h/I_p=0$ (unperturbed map)



$\Delta Y_0=Y_0-Y_{S-}$ where

Y_{S-}=-0,019685

Y_0=initial condition takes from the plate around Y_{S-}. Y_{S-}= the lowest vertical position where the separatrix reaches the plate.

The field line reaches the plate around Y_{S+}.

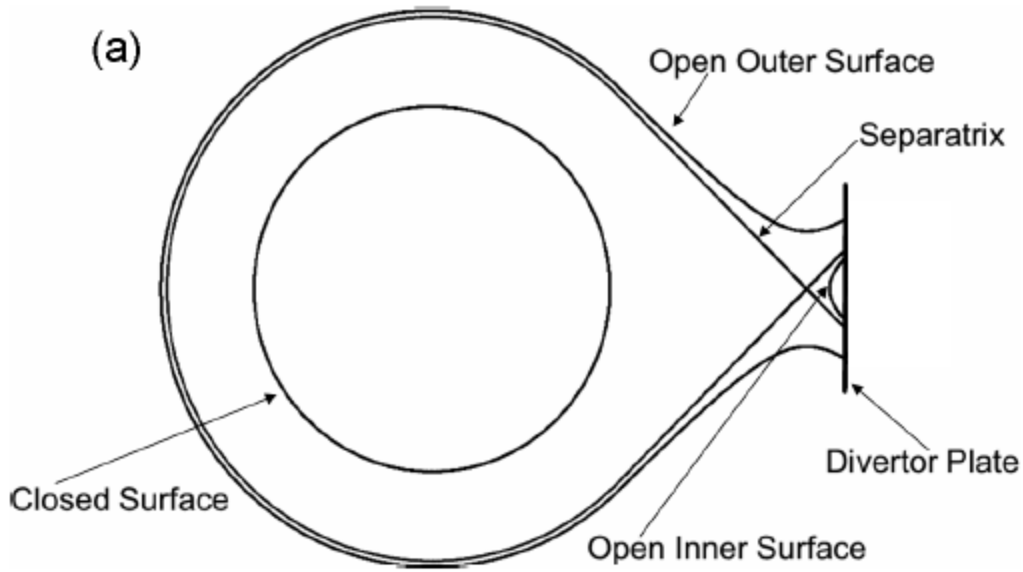
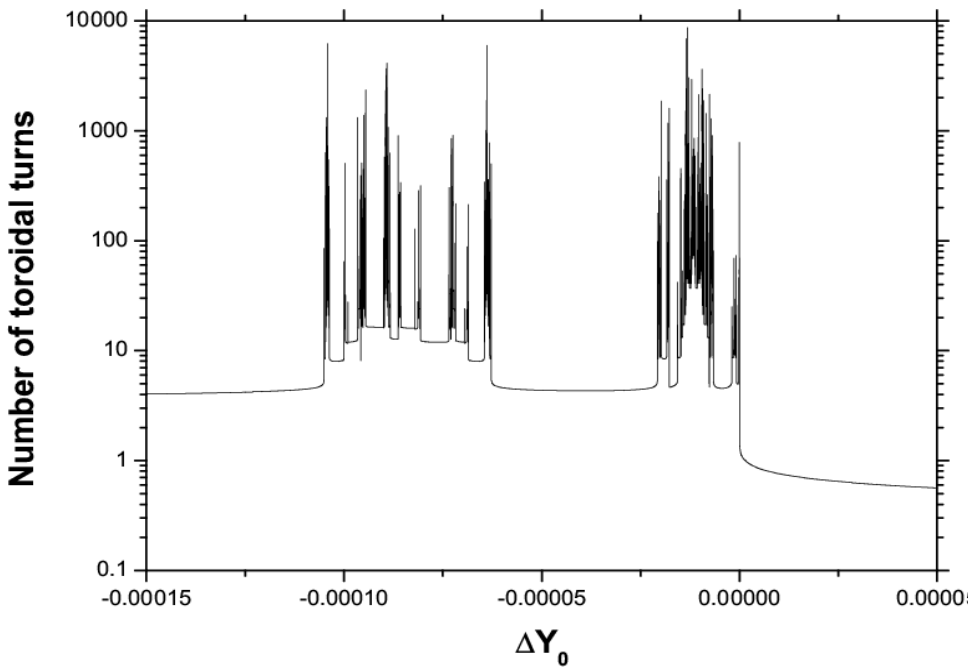


FIG. 2: (a) Scheme of a single-null divertor. (b)

Fractal (continuous) distribution of connection lengths for lines in the chaotic (laminar) region. Reflects details of laminar and chaotic regions.

m=4 I_h/I_p=0.20%



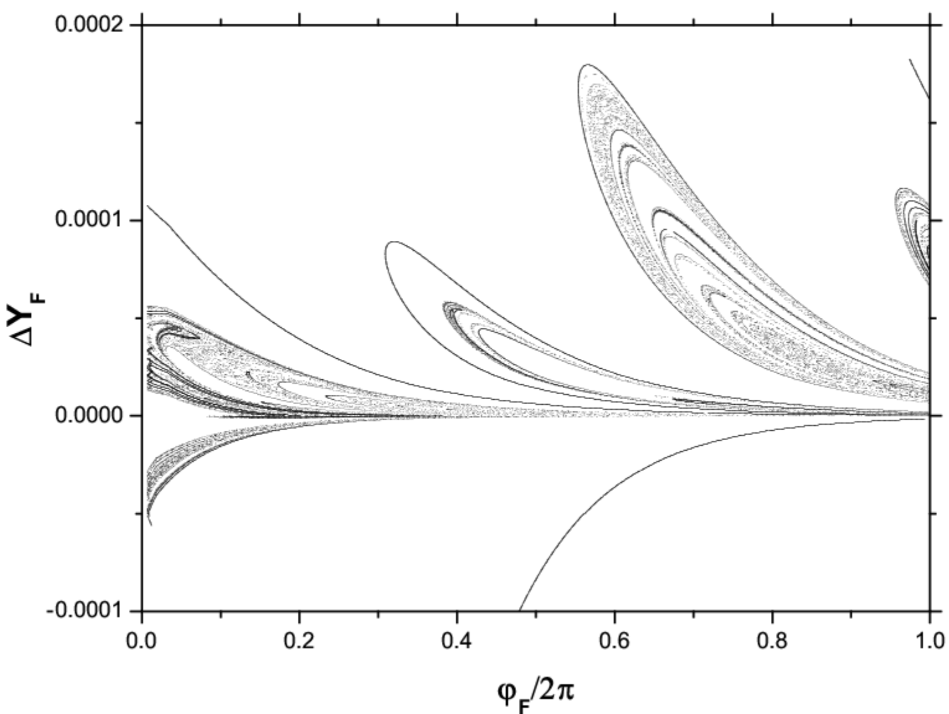
Footprints

$\Delta Y_F = Y_F - Y_{S+}$ where

Y_{S+}=0,019685

Y_{S+}= the highest vertical position where the separatrix reaches the plate.
Y_F=the final vertical coordinate when each line reaches the plate.
φ_F =the final toroidal angular coordinate when each line reaches the plate.

m=4 I_h/I_p=0.20%



φ_F modulate between 0 and 2π:

VI- Conclusions

- A method is used to obtain a symplectic map for field lines in tokamaks with a single null-divertor.
- The map has realistic q-profiles.
- Divertor map perturbed by Martin Taylor map breaks the separatrix and gives rise to an ergodic region.
- Footprints changes with shear and resonant mode.
- Our results can qualitatively reproduce divertor experiments.