“Detection and dynamics of multi-planet systems”

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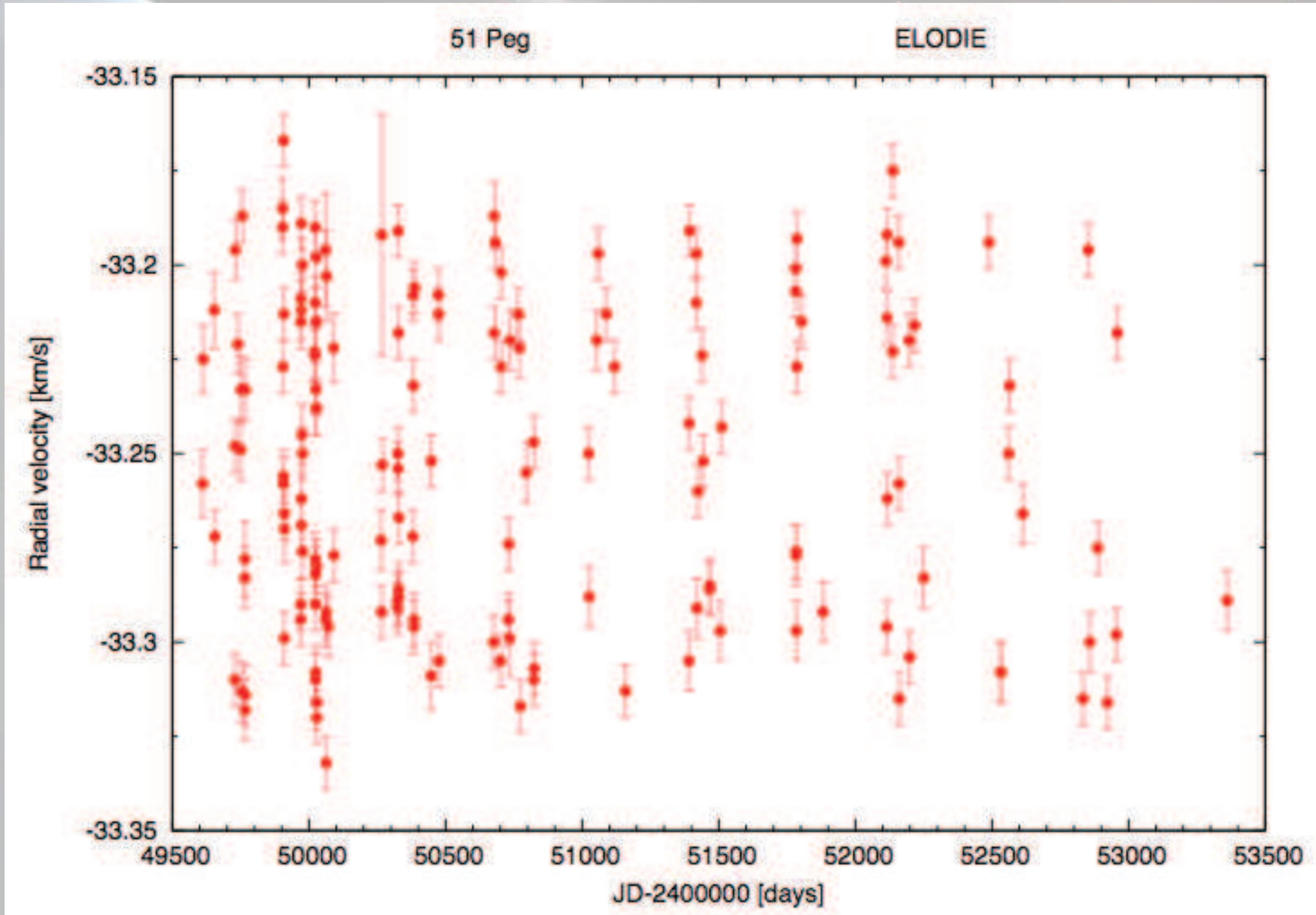
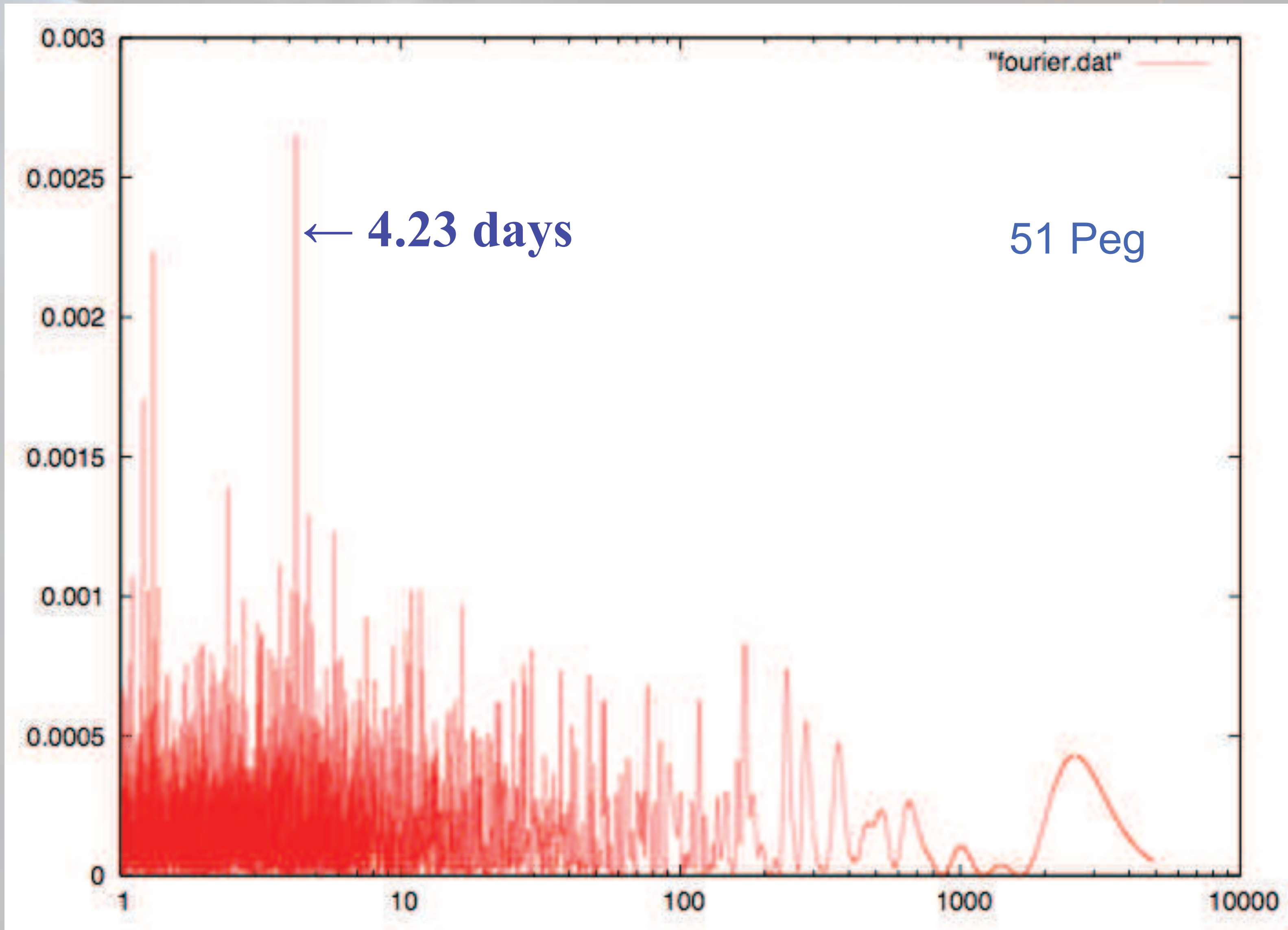
What we REALLY know about exoplanets:

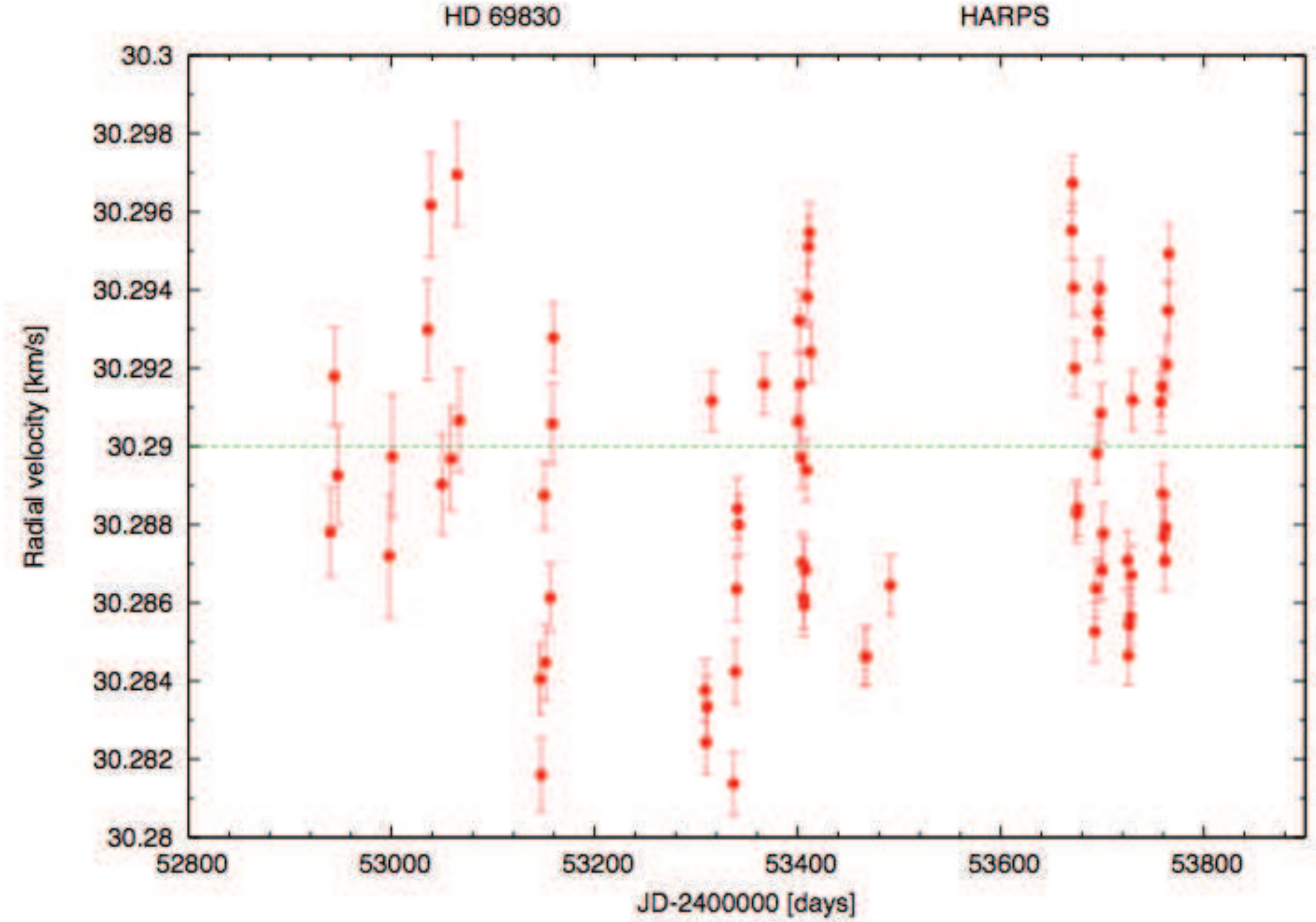


Single planet
Frequency Analysis of Data

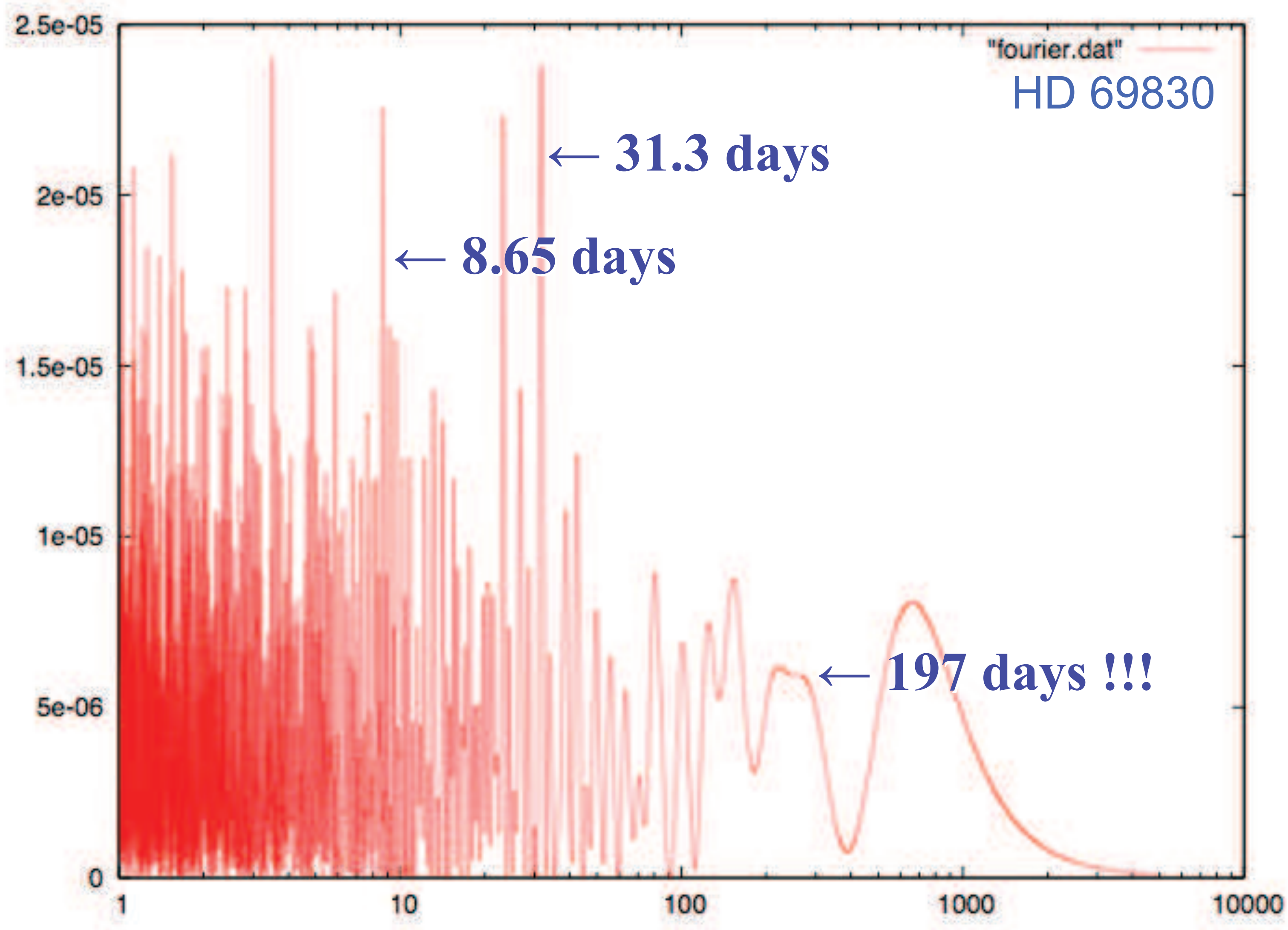


HD 69830
“Neptune’s Trident”

Lovis et al. (2006)



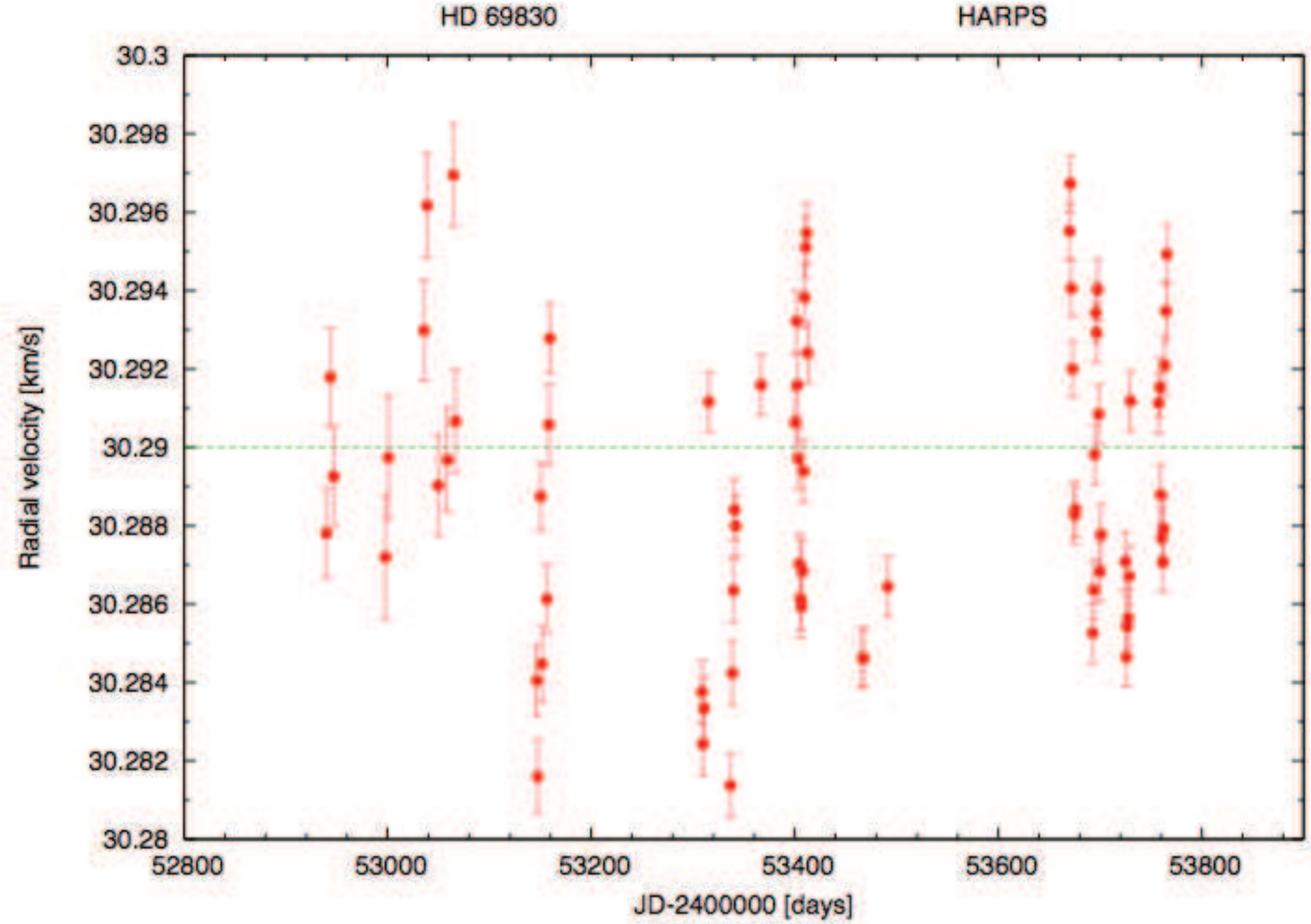
Multiple planets



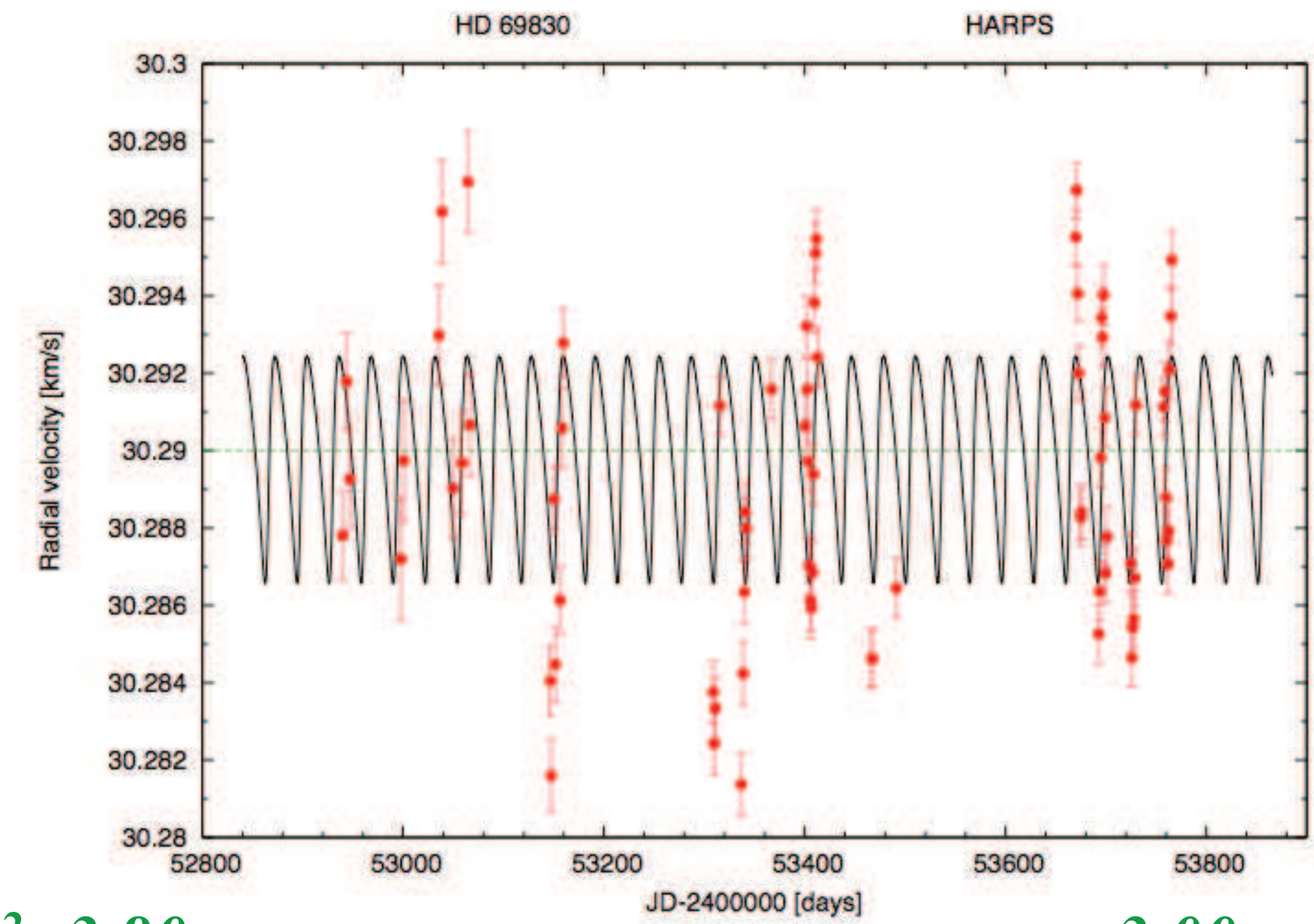
HD 69830

"Neptune's Trident"

Lovis et al. (2006)



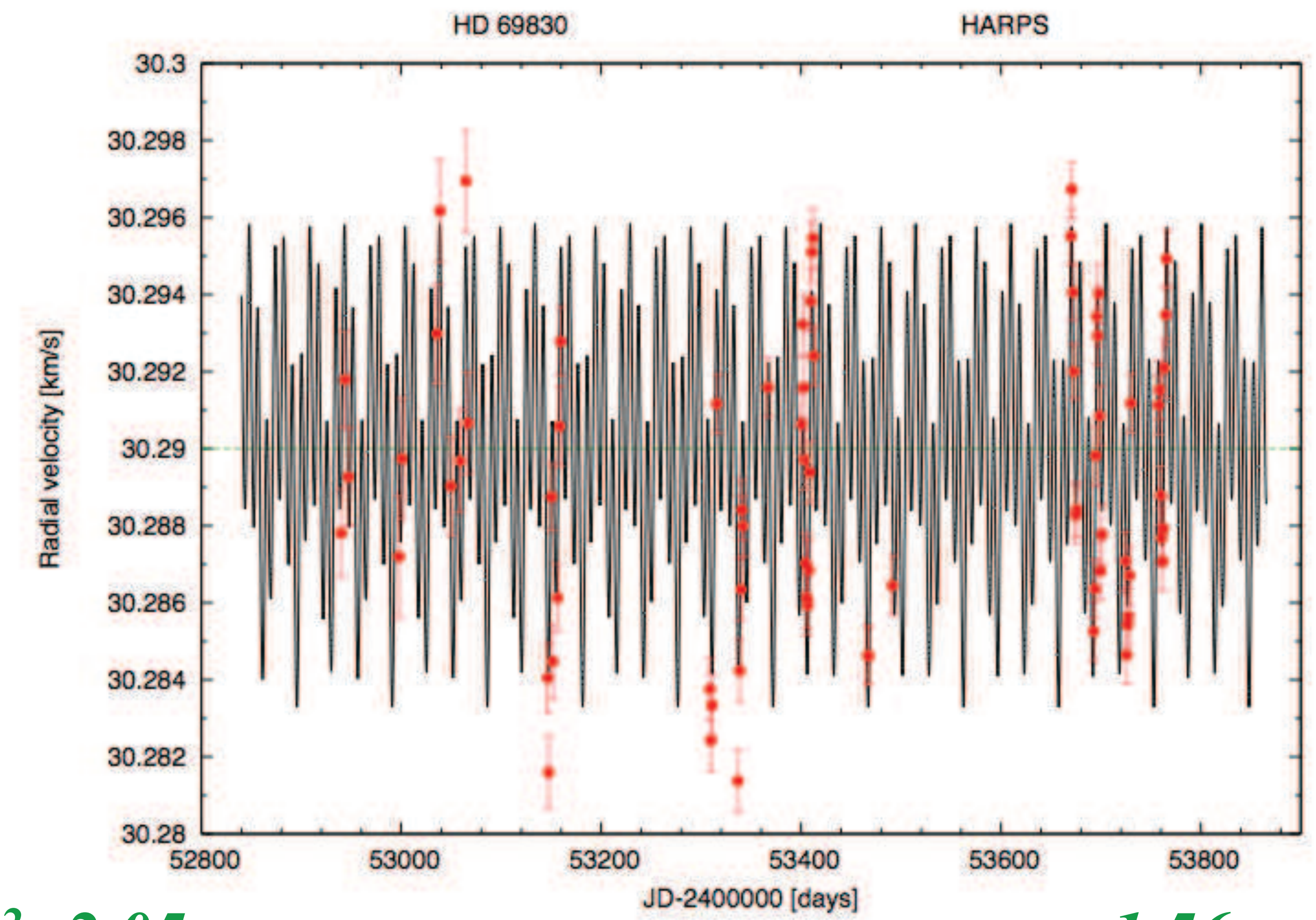
$P_1 = 31.9$ *days* ; $e_1 = 0.35$; $m_1 = 12.4\ M_E$



$\chi^2 = 3.80$ *rms = 3.00 m/s*

$P_2 = 8.66$ days ; $e_2 = 0.04$; $m_2 = 12.0\ M_E$

$P_1 = 31.8$ days ; $e_1 = 0.21$; $m_1 = 11.8\ M_E$



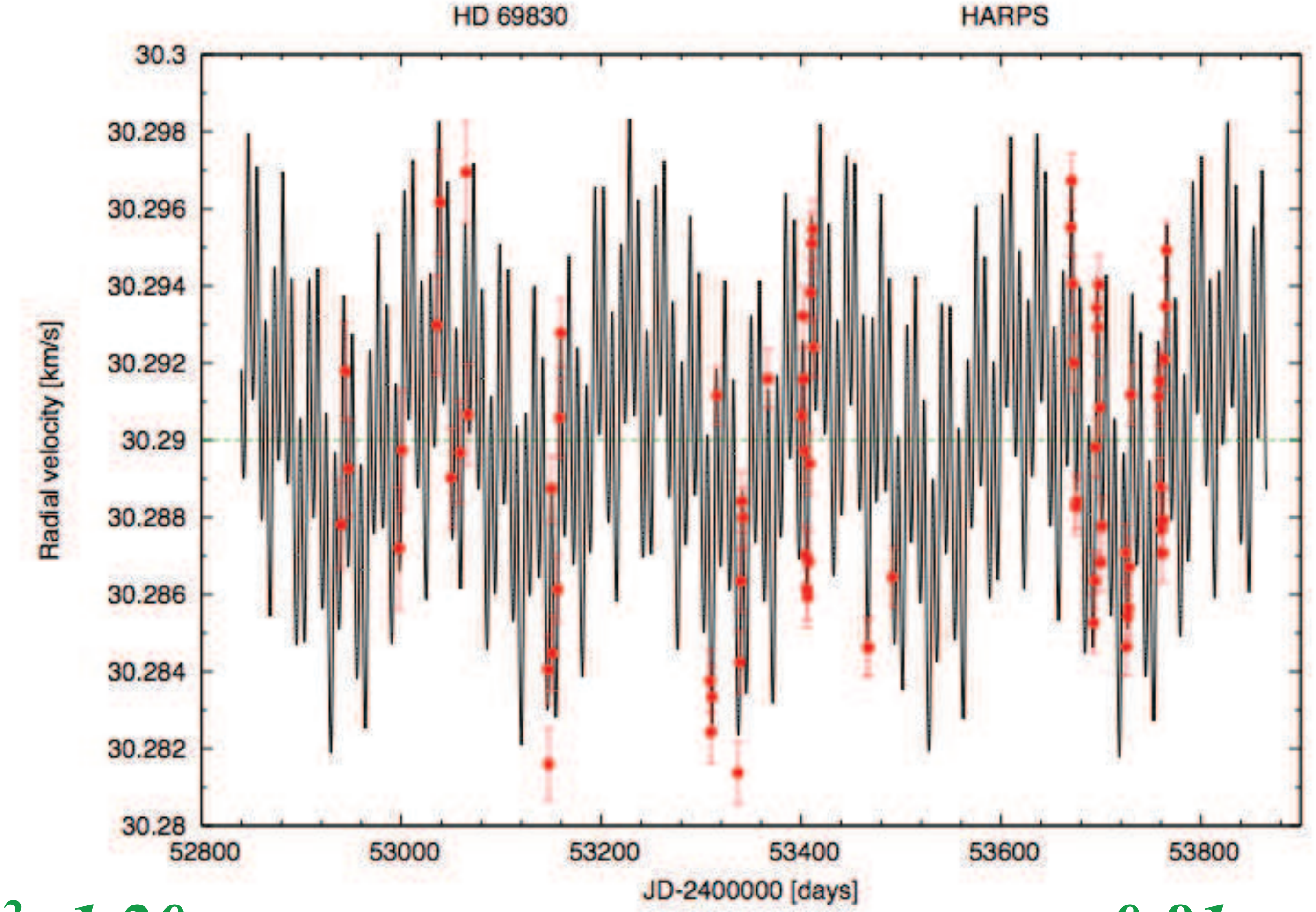
$\chi^2 = 2.05$

rms = 1.56 m/s

$P_2 = 8.67$ days ; $e_2 = 0.10$; $m_2 = 10.2\ M_E$

$P_1 = 31.6$ days ; $e_1 = 0.13$; $m_1 = 11.8\ M_E$

$P_3 = 197$ days ; $e_3 = 0.07$; $m_3 = 18.1\ M_E$



$\chi^2 = 1.20$

$rms = 0.81$ m/s

Two planets motion

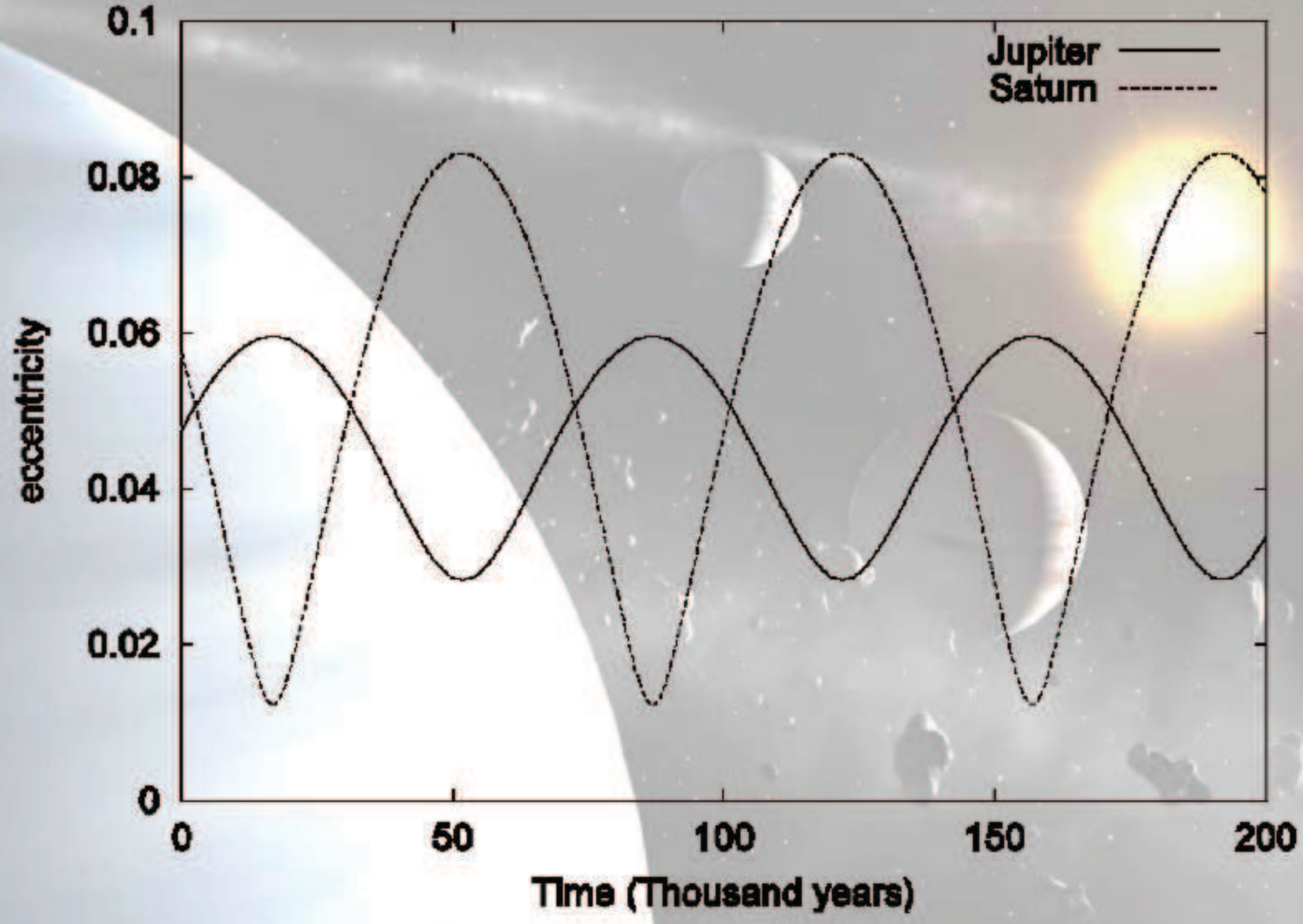
$$\ddot{\vec{r}} = -G\frac{(m+M)}{r^3}\vec{r} + \vec{\nabla}_{\vec{r}}R$$

Disturbing function:

$$R = Gm'\left(\frac{1}{|\vec{r}' - \vec{r}|} - \frac{\vec{r}\cdot\vec{r}'}{r'^3}\right)$$

Since *m'* << *M*, the disturbing function can be seen as a perturbation of the keplerian motion.

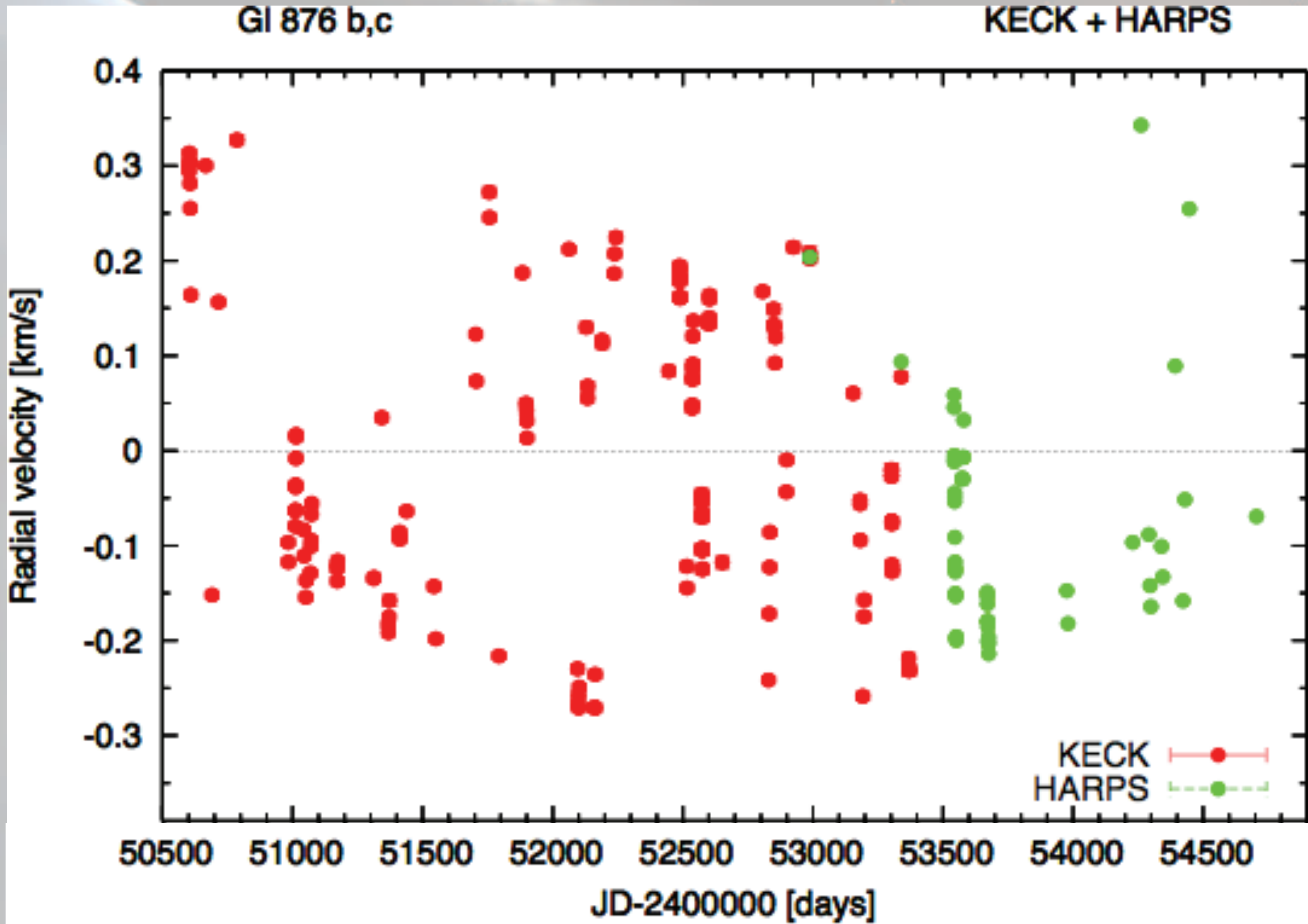
Jupiter and Saturn



Periodicity ~ 70 100 years

GJ 876, a “case study”

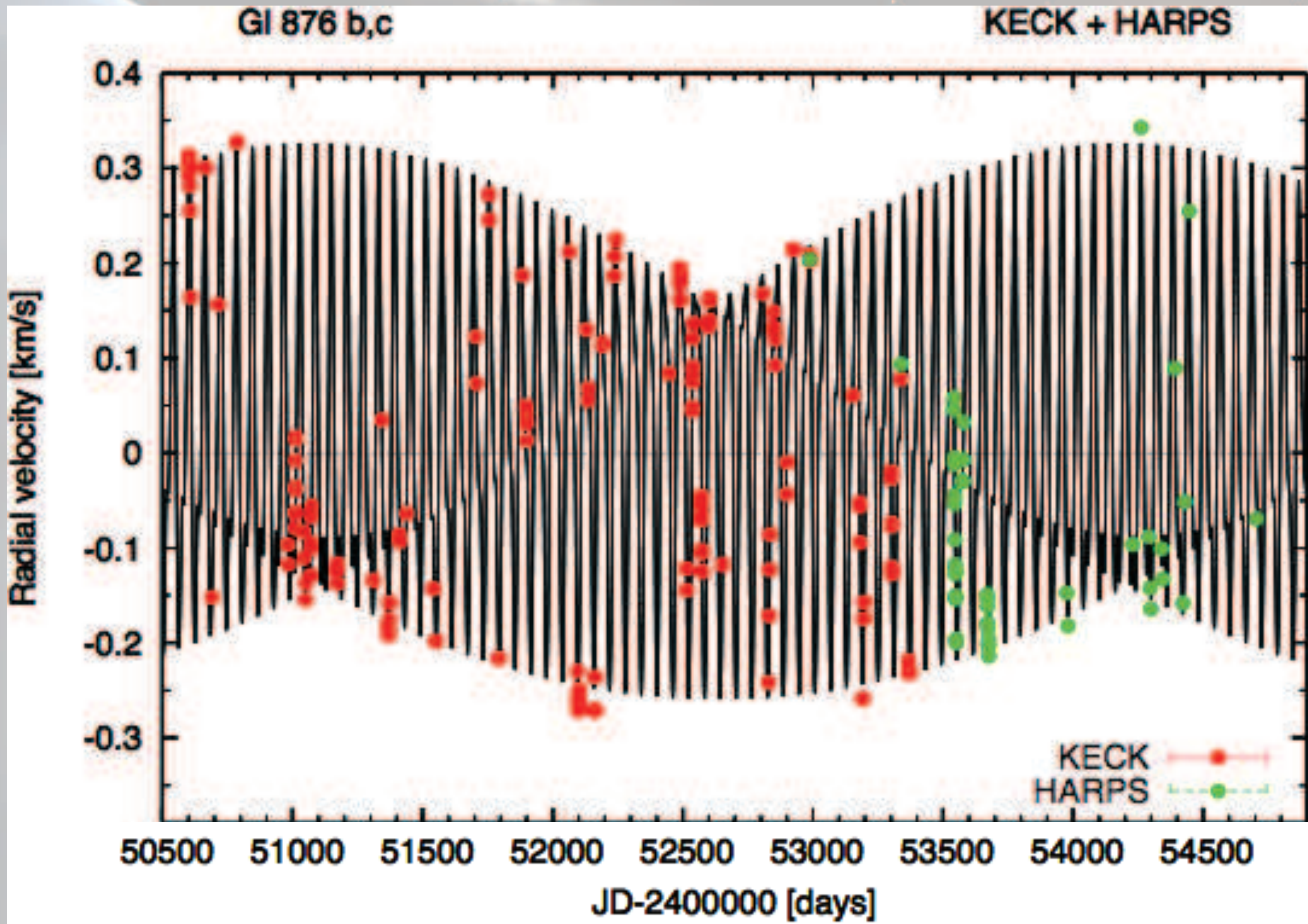
KECK - *HARPS*



Correia et al. (A&A, 2010)

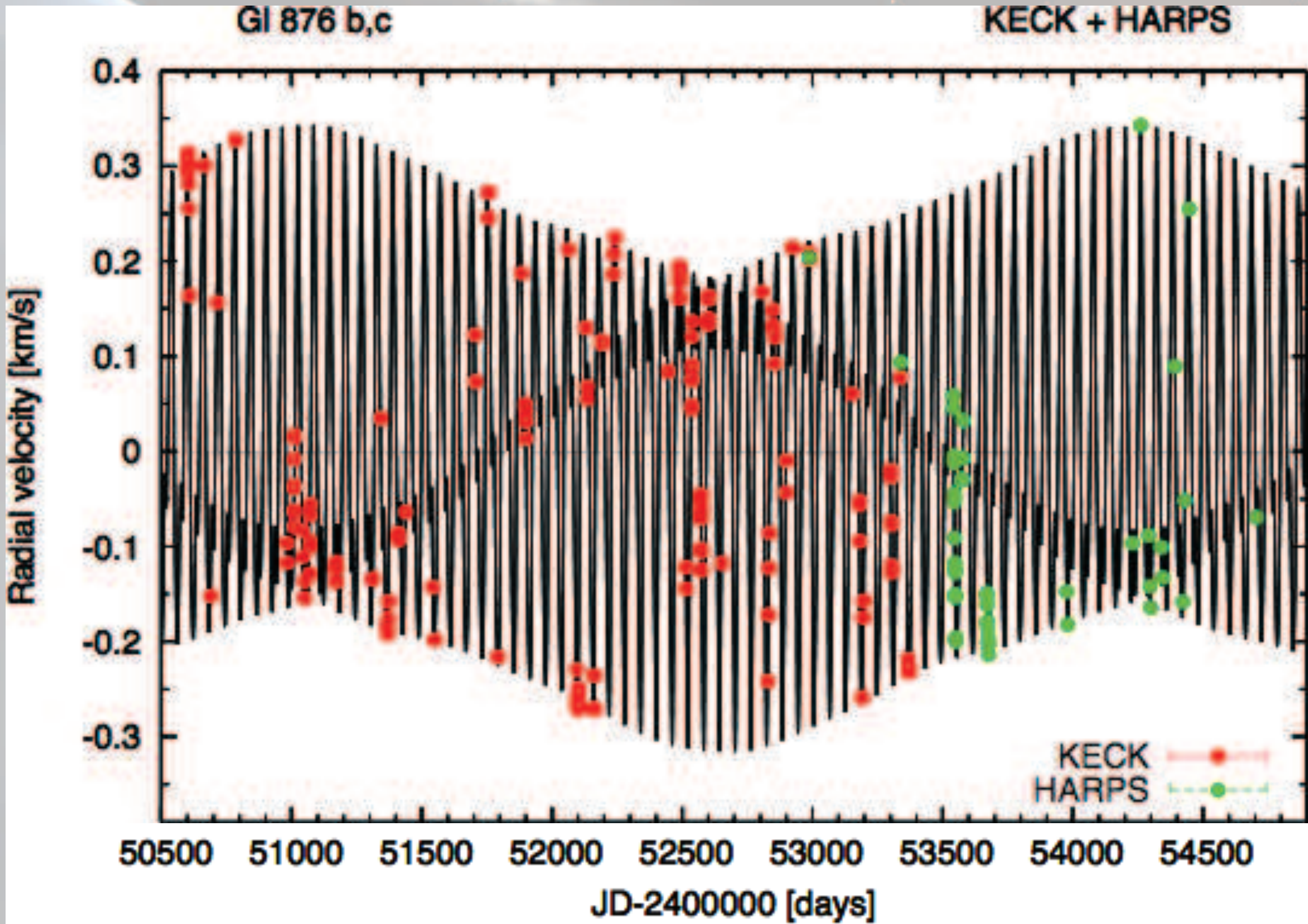
2 planets, two keplerian orbits

$\chi^2=6.41$ $i = ??$ $rms = 11.0\ m/s$



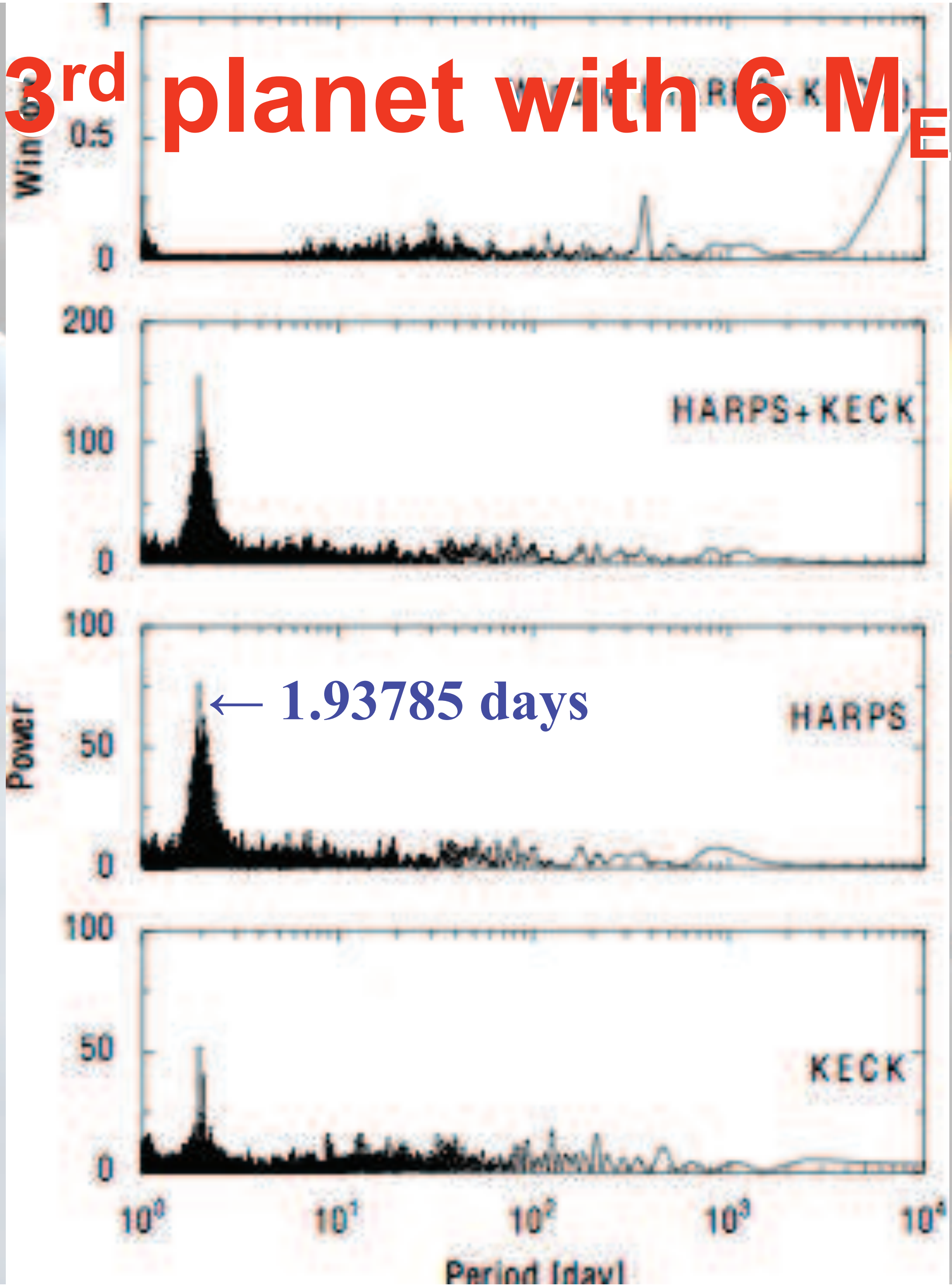
2 planets, planet-planet interaction

$\chi^2 = 2.60$ $i = 49^o \pm 1^o$ $rms = 4.45\ m/s$



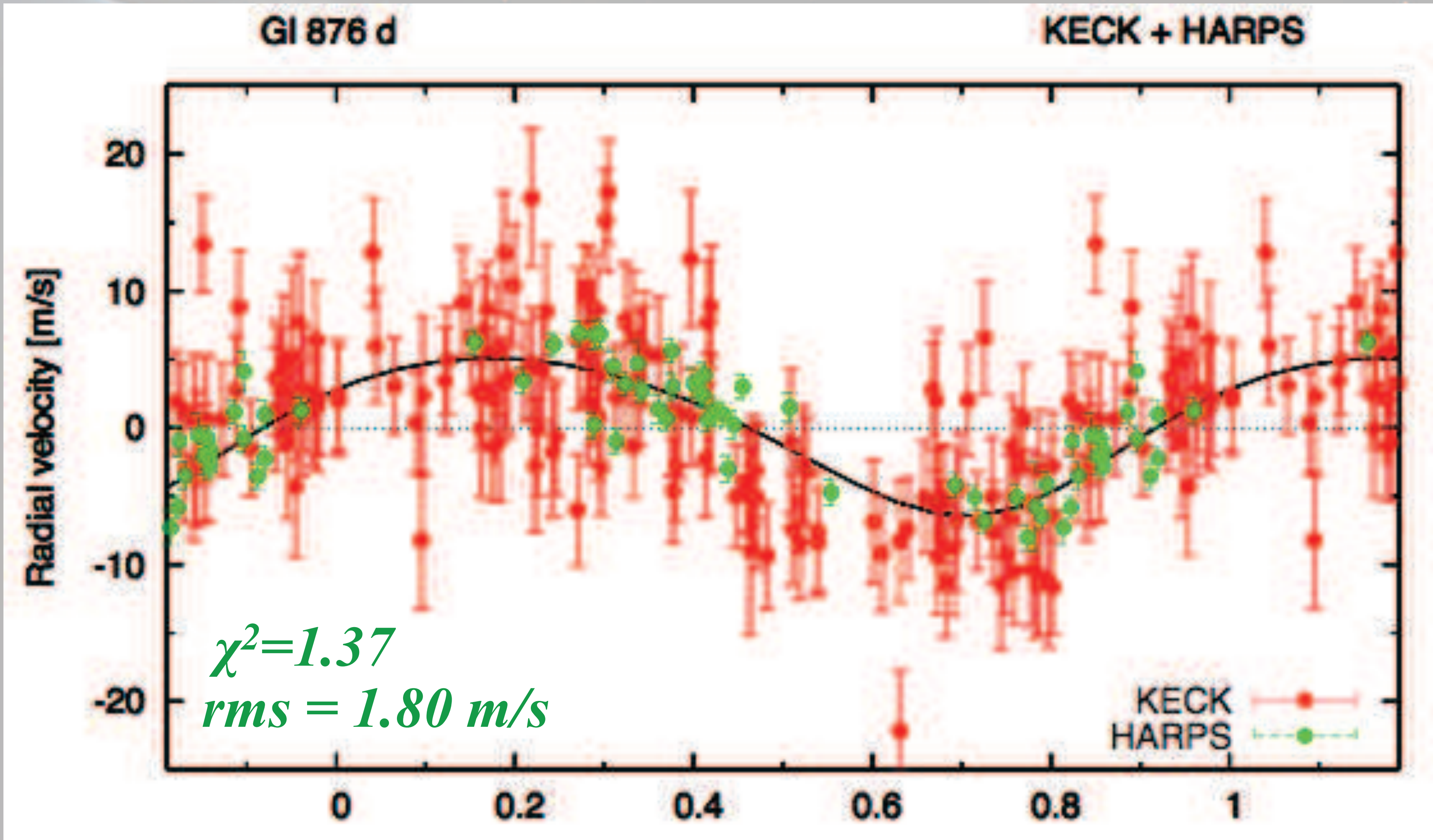
Correia et al. (A&A, 2010)

3rd planet with 6 M_E



Correia et al. (A&A, 2010)

3^{rd} planet with 6 M_E



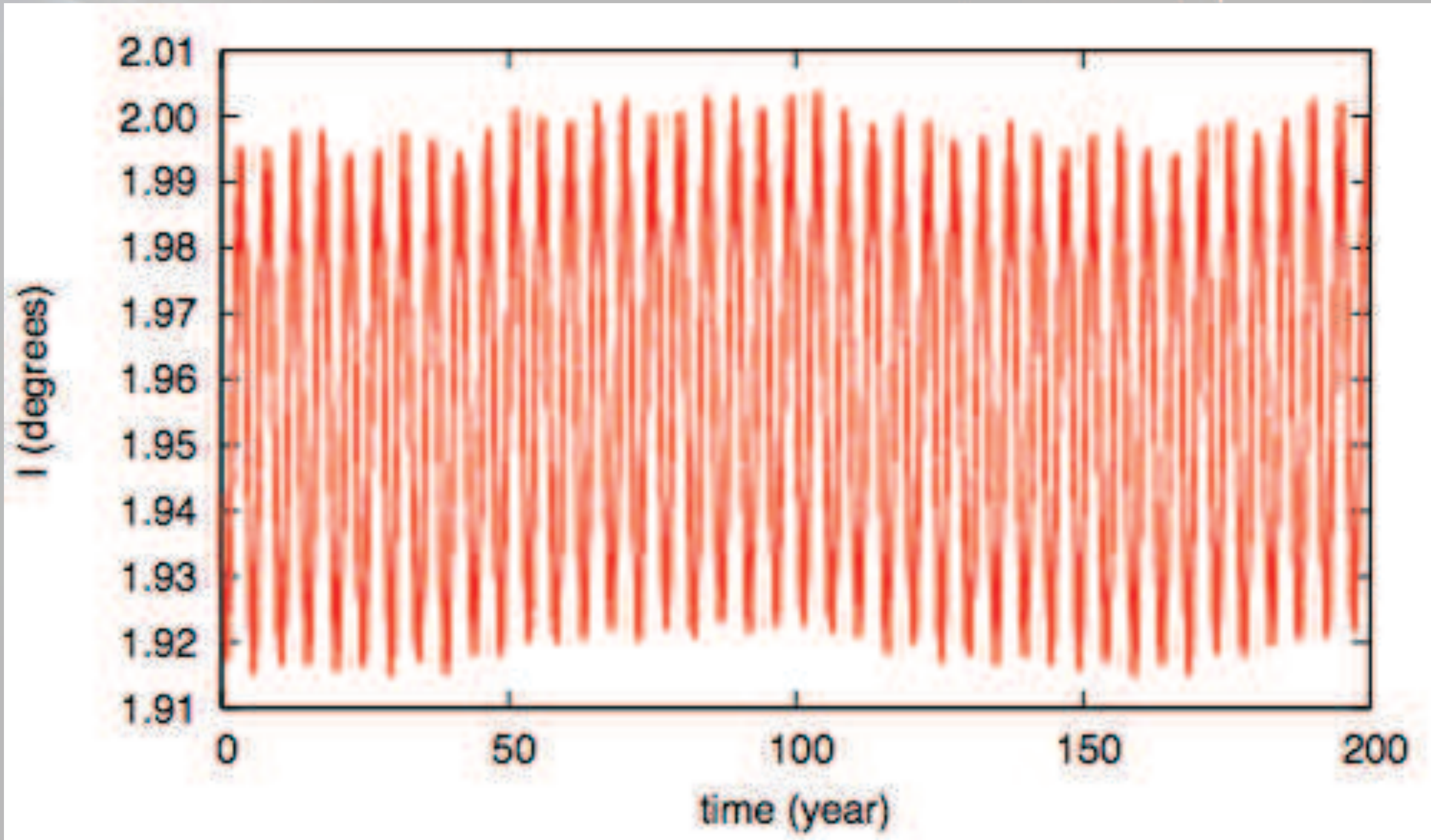
$\chi^2=1.37$

rms = 1.80 m/s

Correia et al. (A&A. 2010)

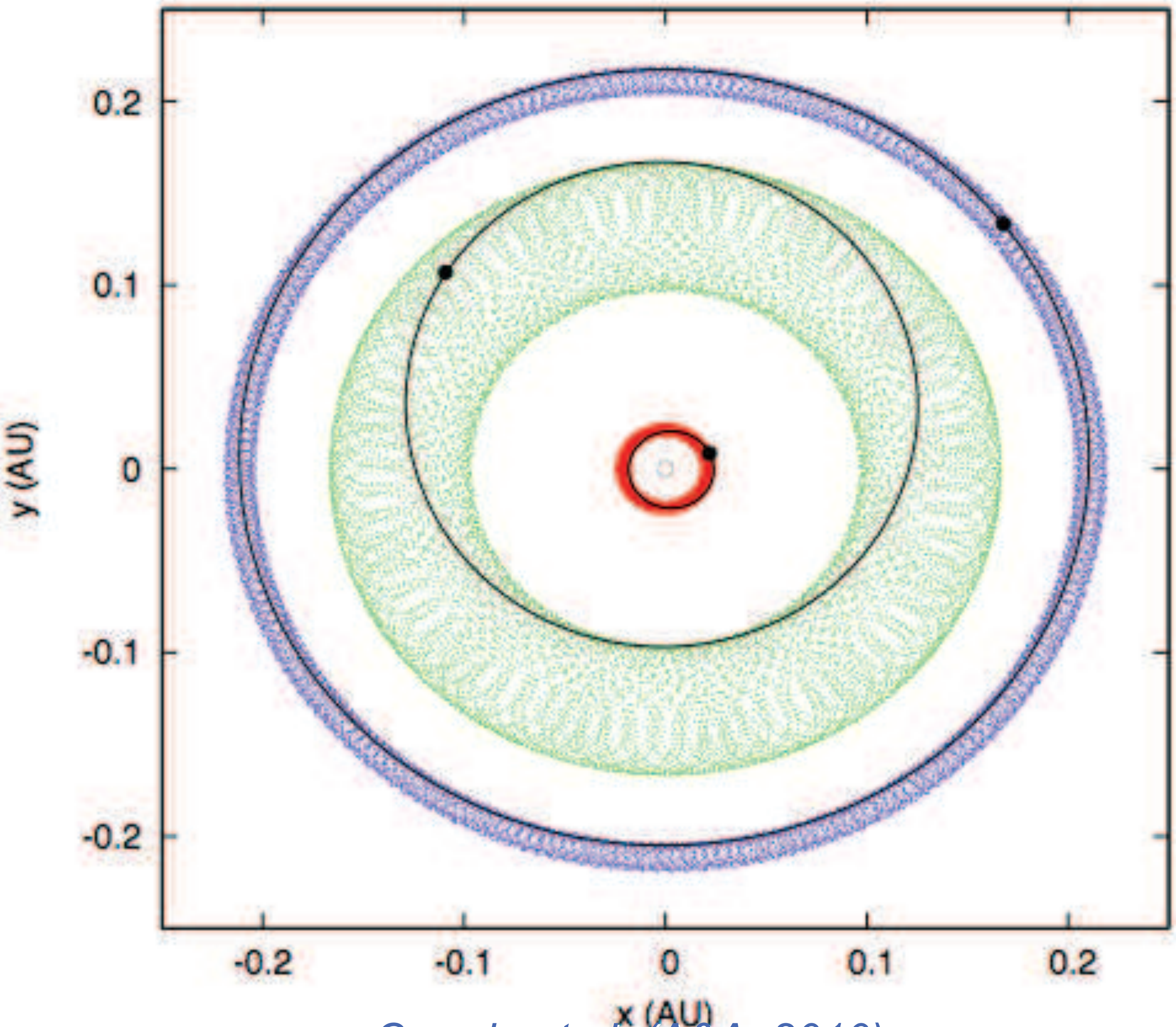
Param.	[unit]	b	c	d
Date	[JD]		2 455 000.00 (fixed)	
$V_{(KECK)}$	[km/s]		0.0130 ± 0.0004	
$V_{(HARPS)}$	[km/s]		−1.3388 ± 0.0004	
P	[day]	61.067 ± 0.011	30.258 ± 0.009	1.93785 ± 0.00002
λ	[deg]	35.61 ± 0.14	158.62 ± 0.80	29.94 ± 3.30
e		0.029 ± 0.001	0.266 ± 0.003	0.139 ± 0.032
ω	[deg]	275.52 ± 2.67	275.26 ± 1.25	170.60 ± 15.52
K	[m/s]	212.24 ± 0.33	86.15 ± 0.40	6.67 ± 0.26
i	[deg]	48.93 ± 0.97	48.07 ± 2.06	50 (fixed)
Ω	[deg]	0 (fixed)	−2.32 ± 0.94	0 (fixed)
$a_1 \sin i$	[10^{-3} AU]	1.19	0.23	1.2×10^{-3}
$f(M)$	[10^{-9} M]	60.41	1.79	5.8×10^{-5}
$M \sin i$	[M]	–	–	6.3
M	[M_{Jup}]	2.64	0.83	–
a	[AU]	0.211	0.132	0.021
N_{meas}			207	
Span	[day]		4103	
$\overline{\chi^2}$			1.37	
$rms_{(KECK)}$	[m/s]		4.25	
$rms_{(HARPS)}$	[m/s]		1.80	

Coplanar motion



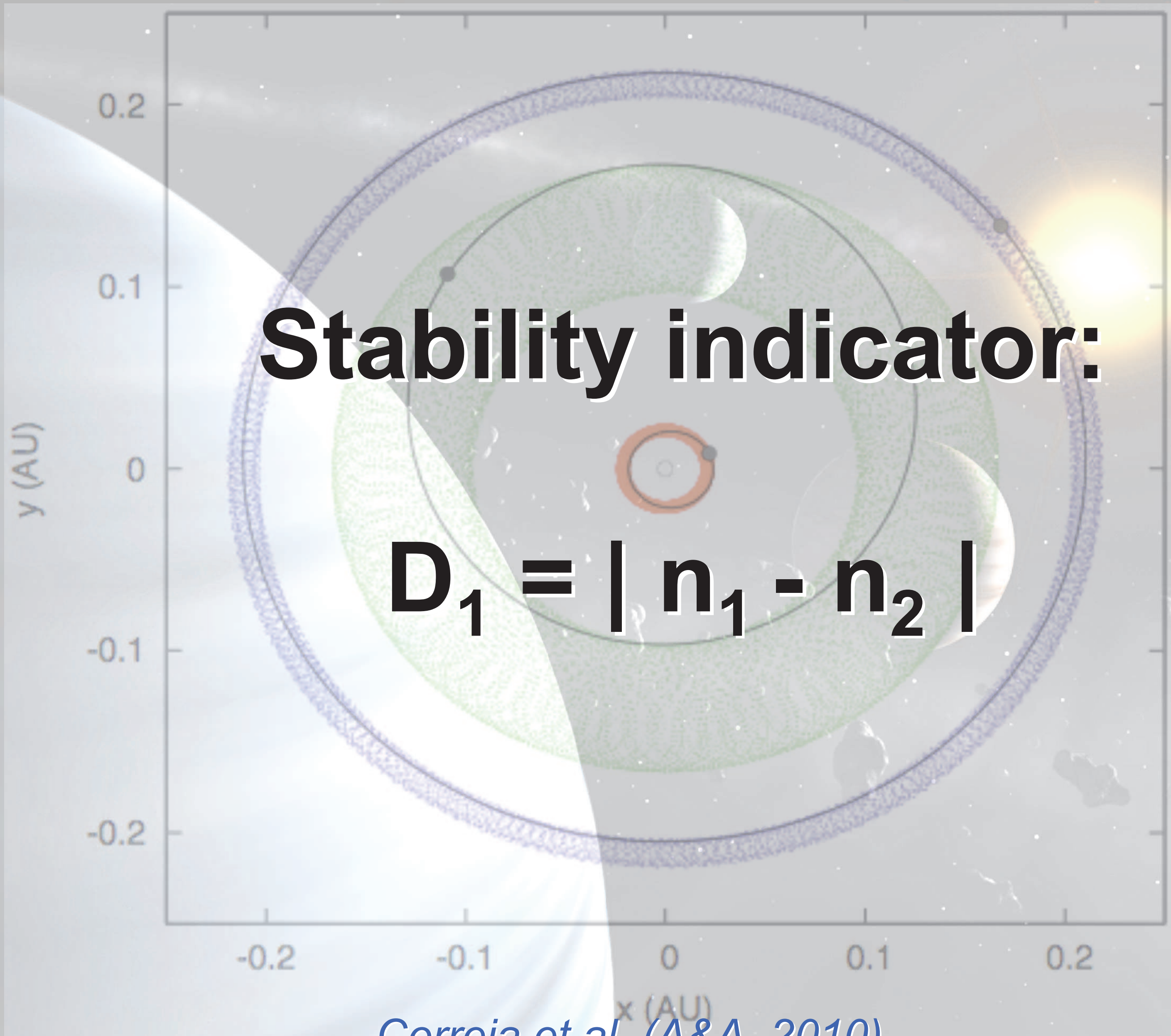
Correia et al. (A&A. 2010)

Long-term stability



Correia et al. (A&A. 2010)

Long-term stability

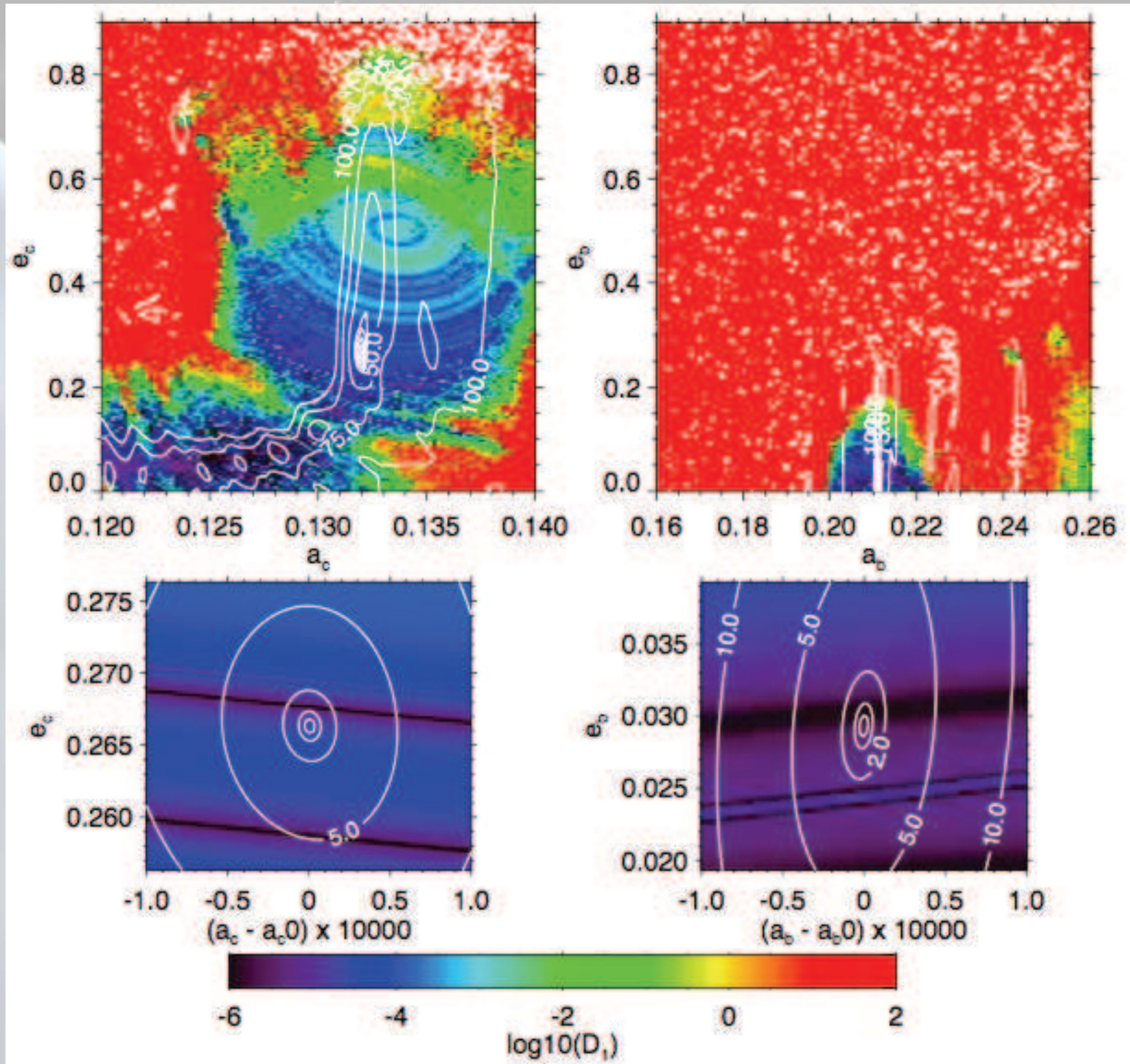


Stability indicator:

$$D_1 = | n_1 - n_2 |$$

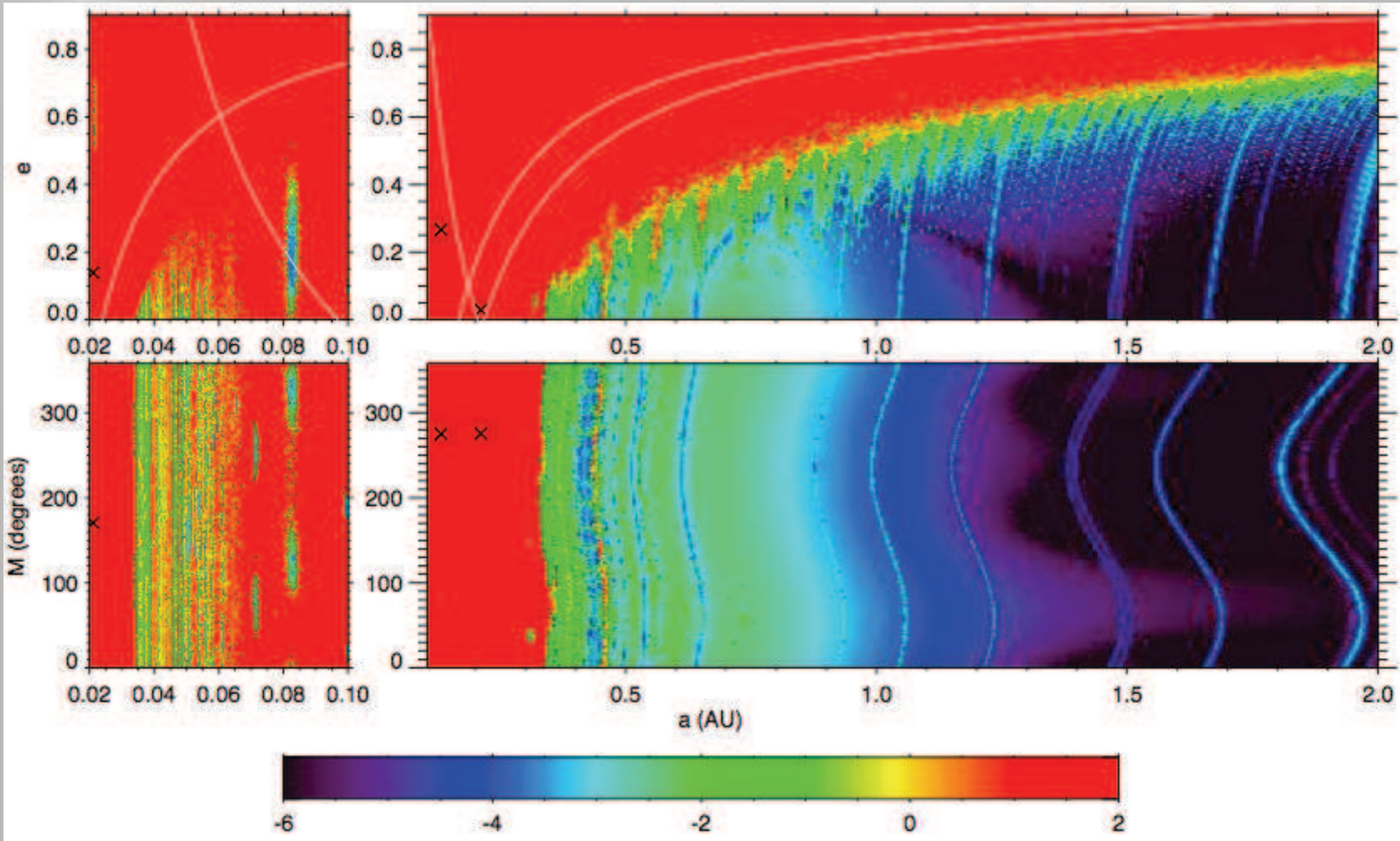
Correia et al. (A&A. 2010)

2:1 mean motion resonance



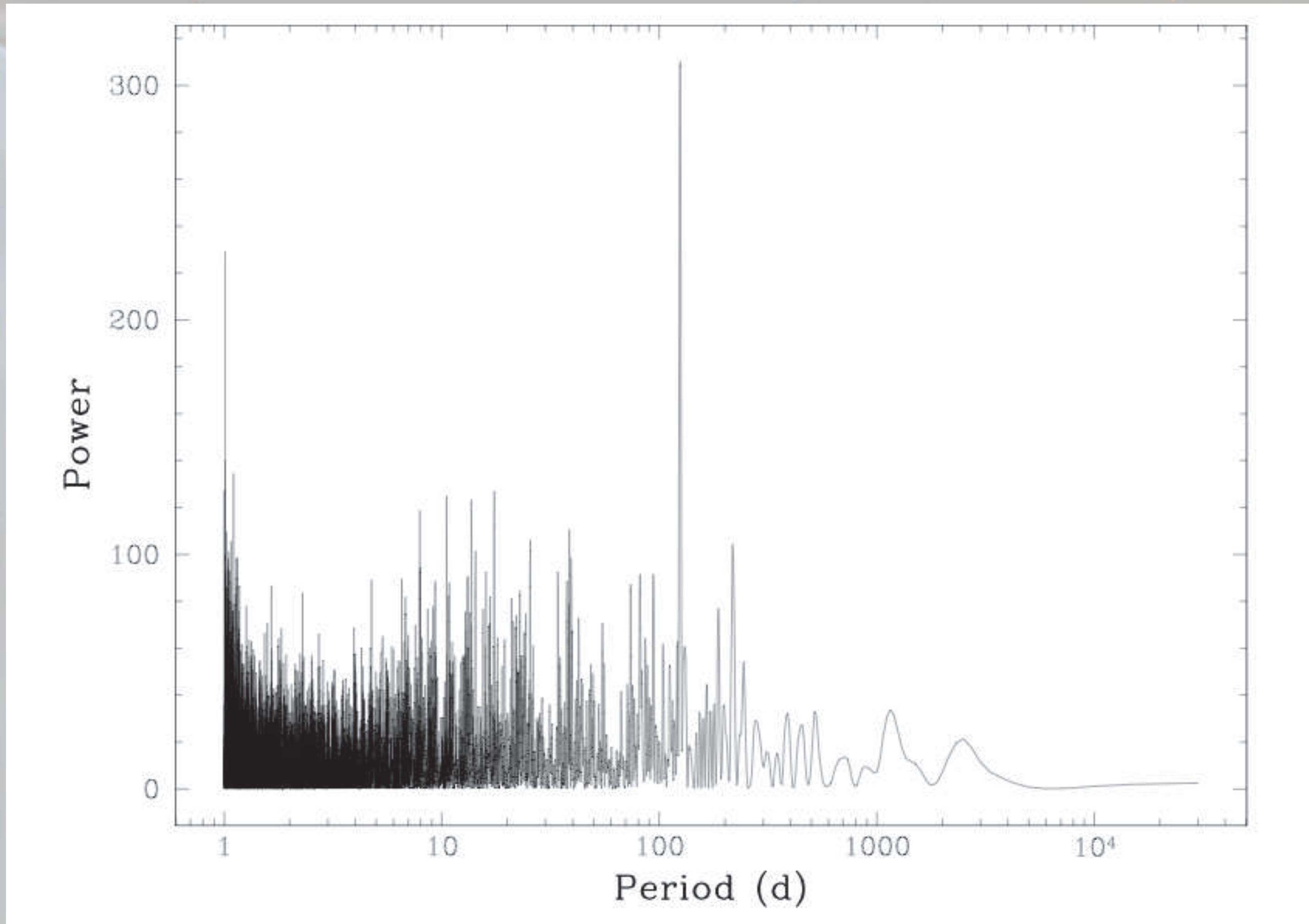
Correia et al. (A&A. 2010)

Additional planets?



Correia et al. (A&A. 2010)

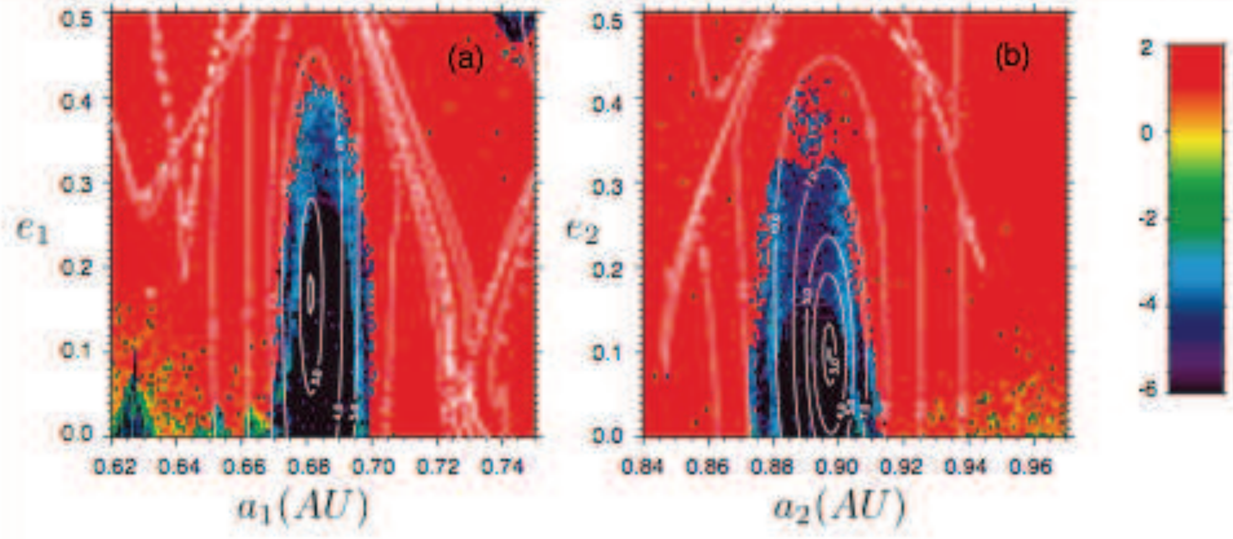
Additional planets?



P = 127 days, 12 Me, Laplace resonance!

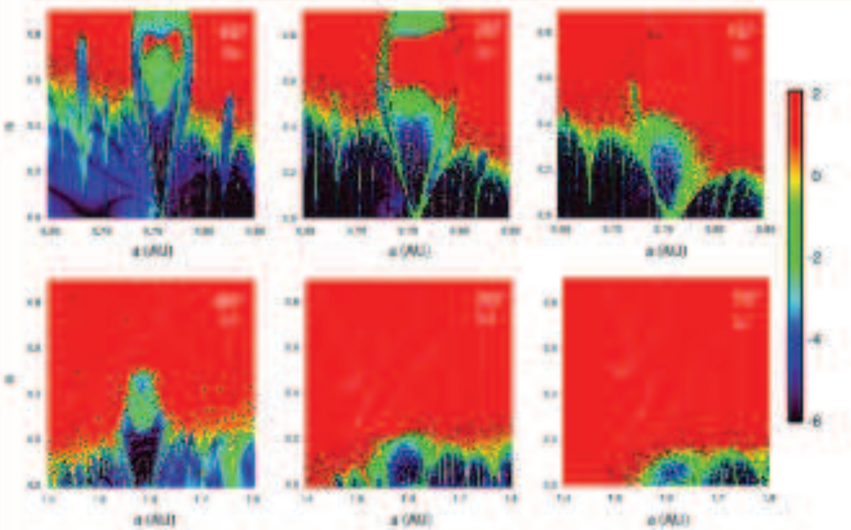
Rivera et al. (Ap J 2010)

HD45364 : first system in 3:2 resonance



Correia, Udry, Mayor, Benz, Bertaux, Bouchy, Laskar, Lovis, Mordasini, Pepe, Queloz, A&A, 2009

HD60532 : System in 3:1 resonance



The occurence of systems in resonance can constraint the dissipation in the formation process

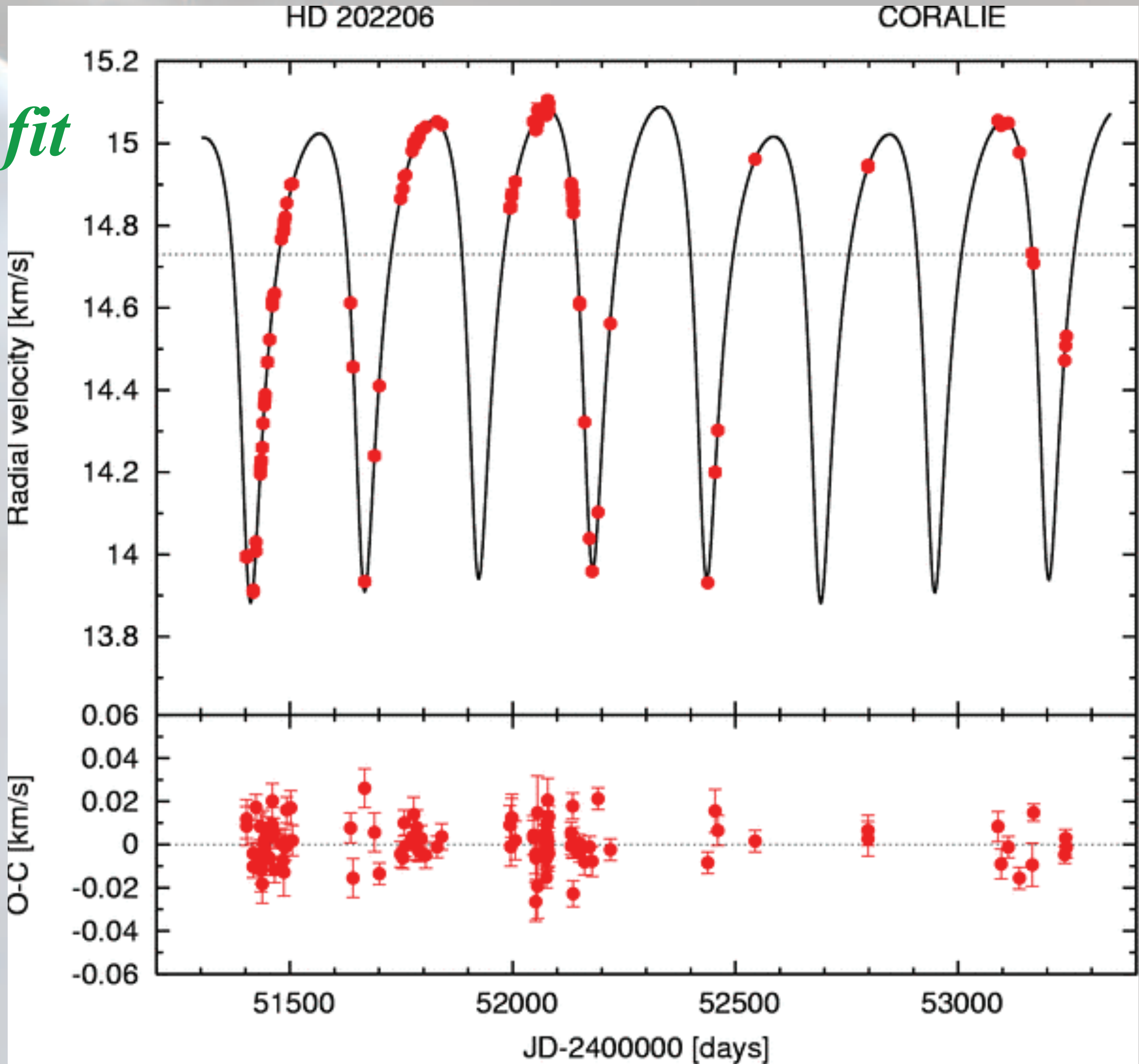
Laskar & Correia, A&A, 2009

2 planets, transition situation HD202206 (5:1 resonance)

keplerian fit

χ^2=1.53

rms =
9.81 m/s



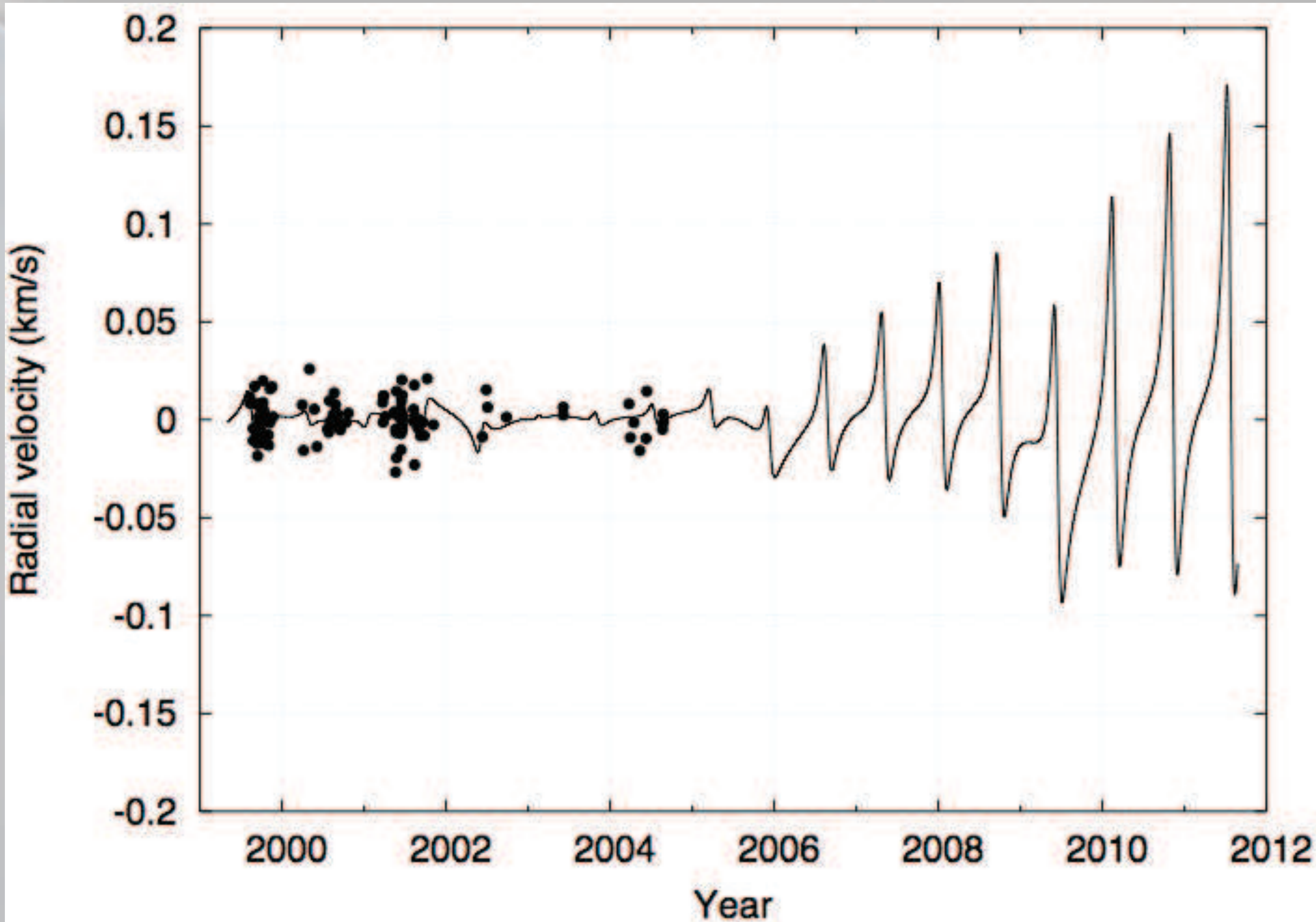
N-body fit

χ^2=1.47

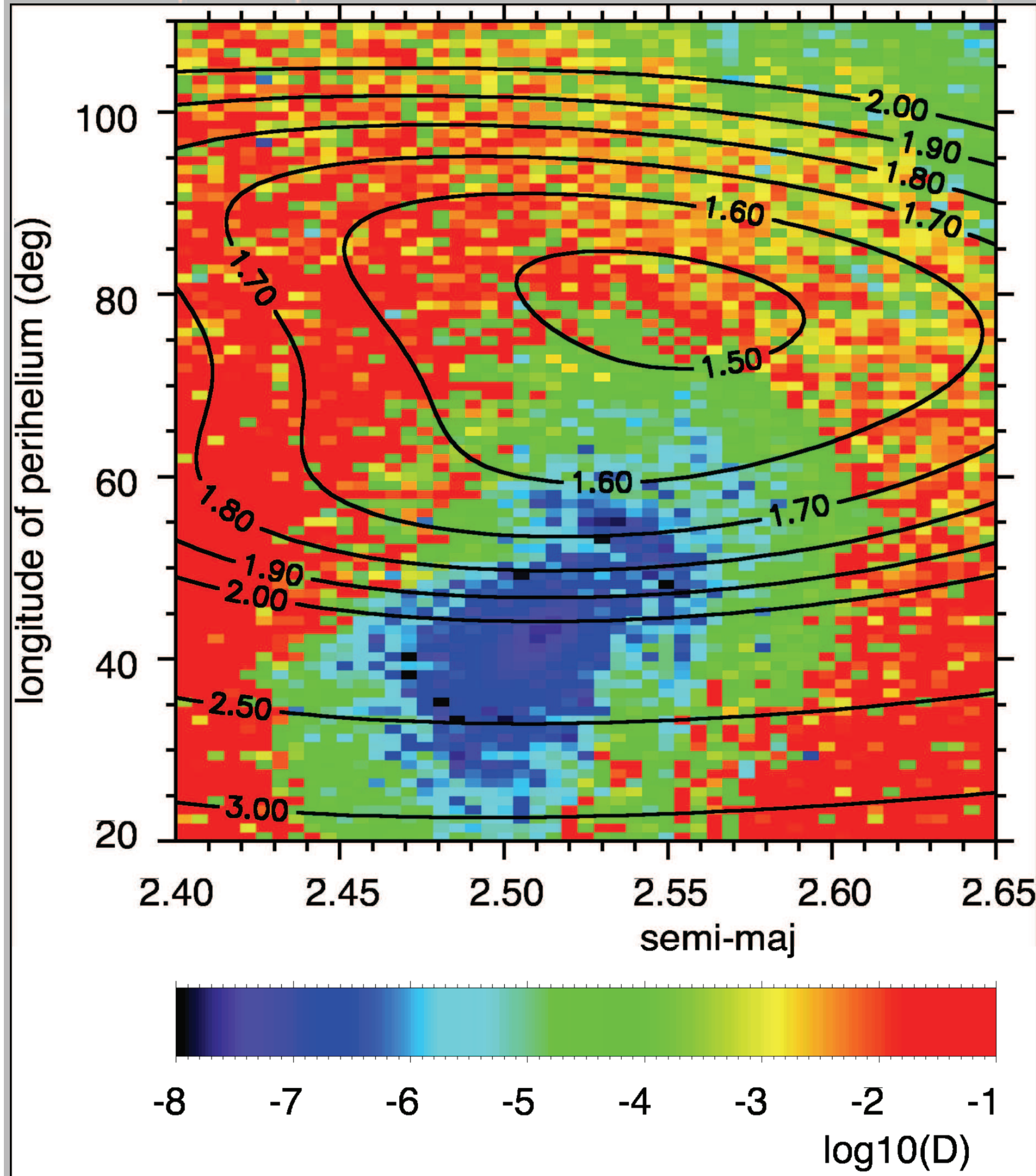
rms =
9.65 m/s

Correia et al. (2005)

Difference between fits



Correia et al. (2005)



a = 2.55 AU

ω = 79.0º

χ^2=1.47

rms = 9.65 m/s

a = 2.54 AU

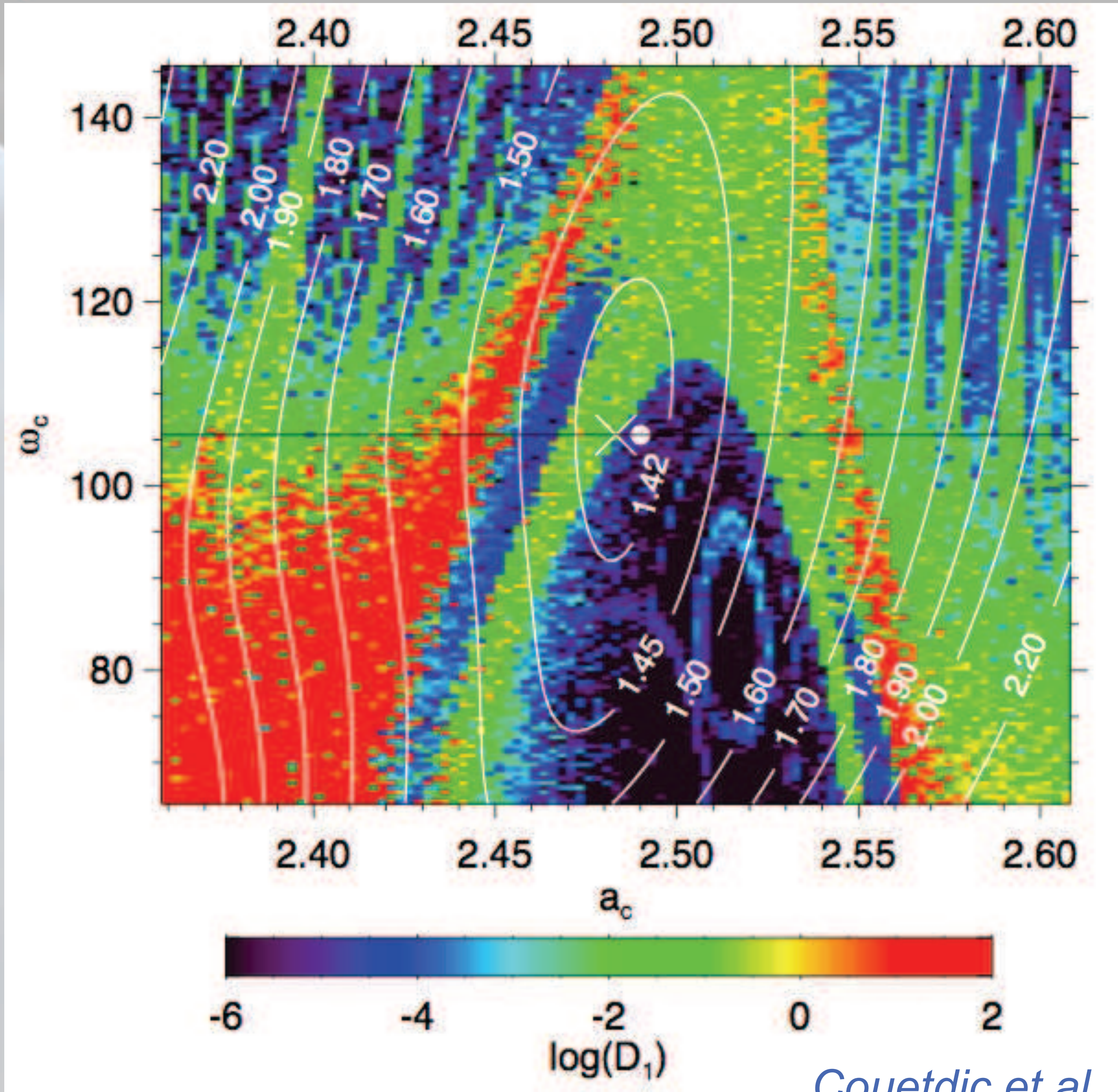
ω = 55.5º

χ^2=1.67

rms = 10.7 m/s

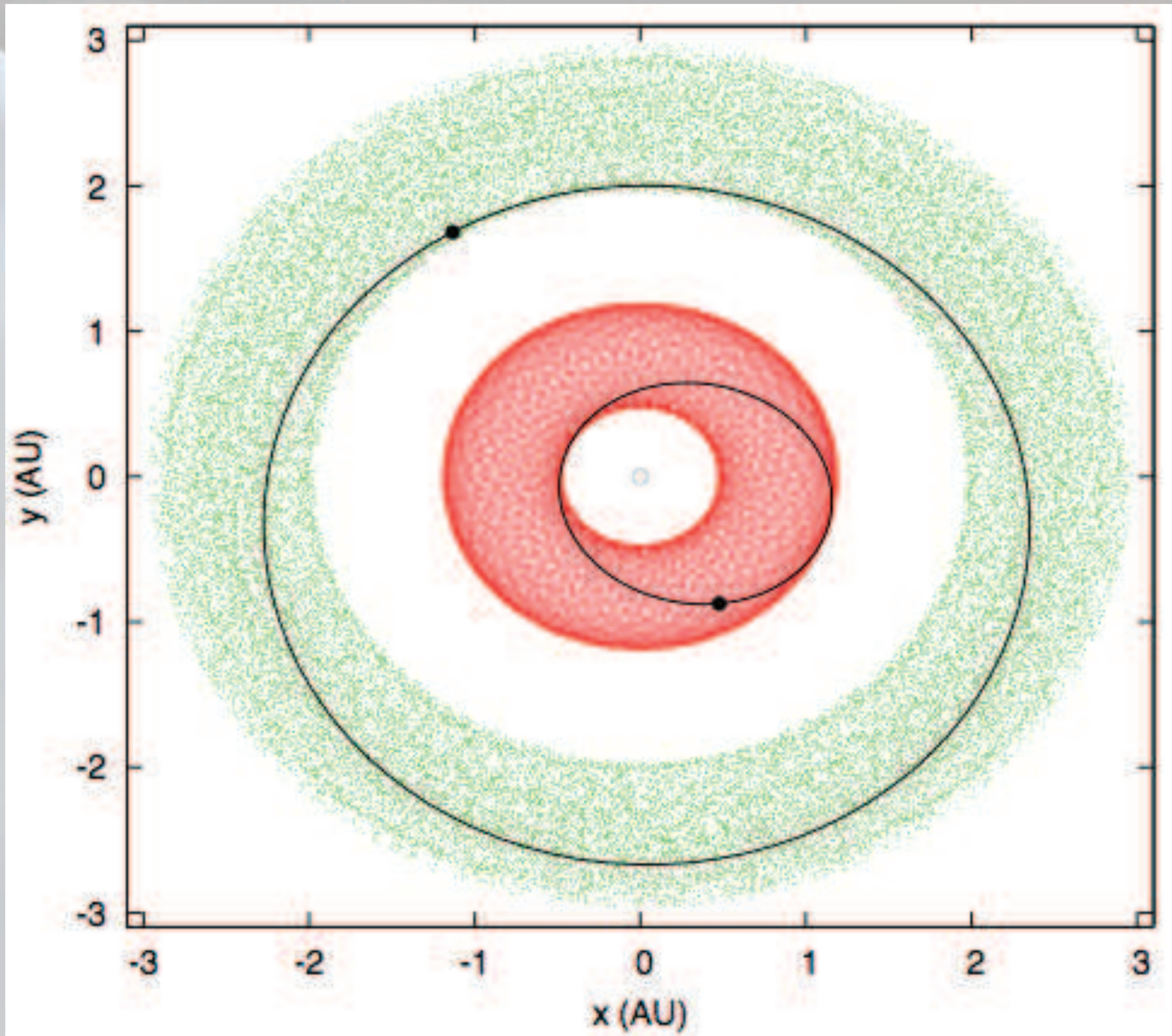
Correia et al. (2005)

HD202206 (2010 updated data)

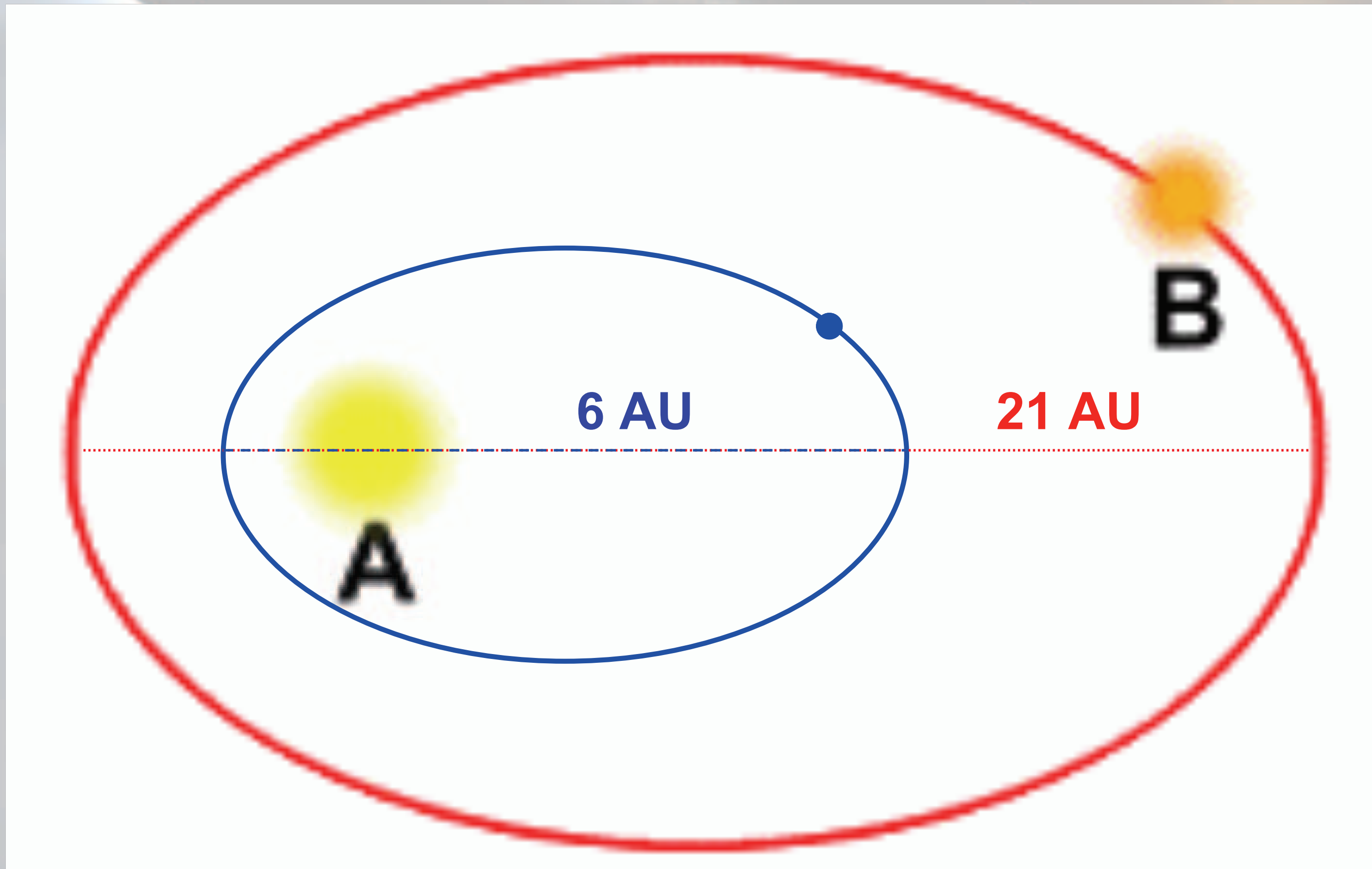


Couetdic et al. (A&A 2010)

Stability over 5 Gyr



HD 196885
close binary system



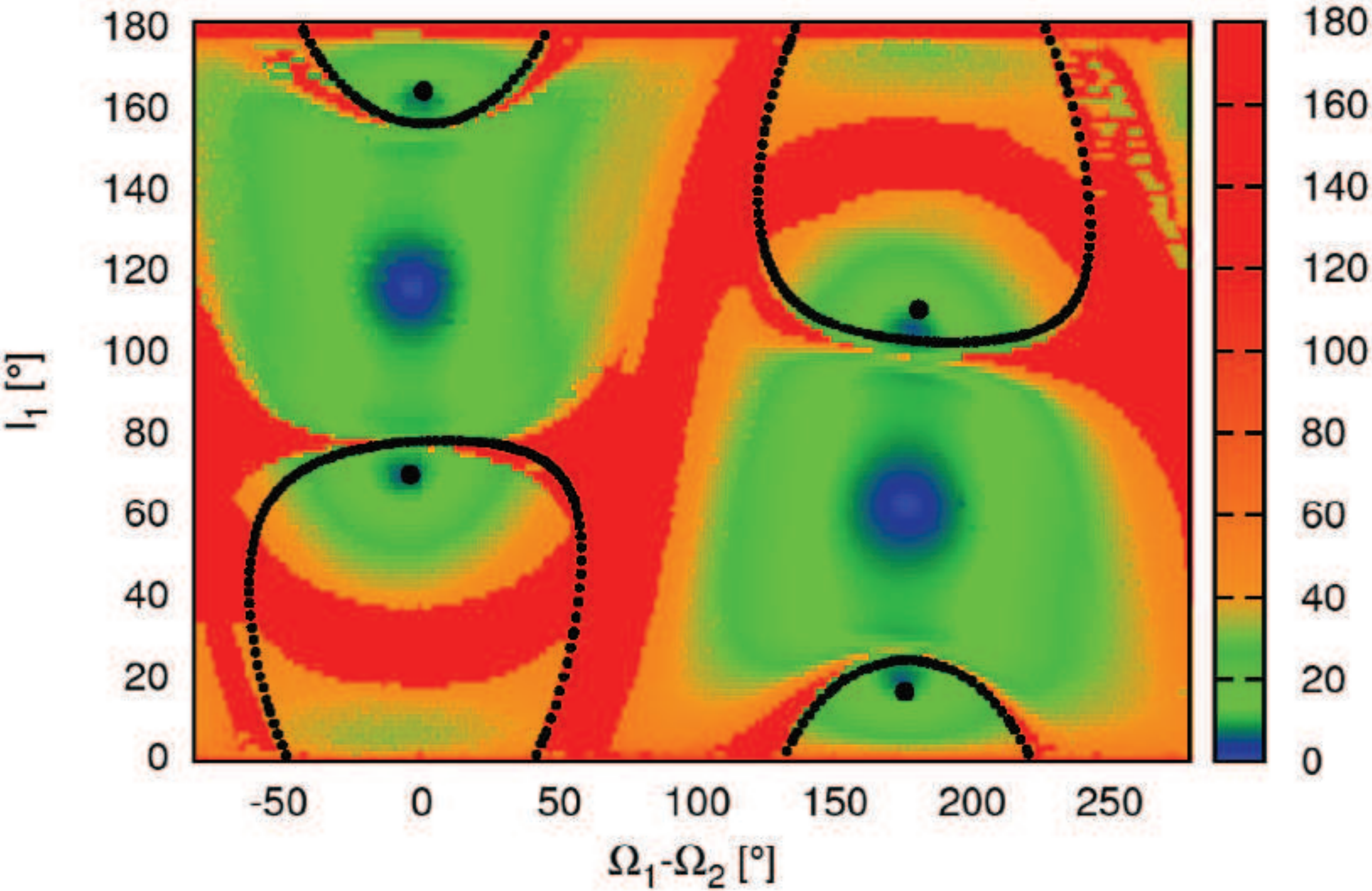
Correia et al. (A&A 2008), Chauvin et al. (A&A 2012)

HD 196885
close binary system

Param.	[unit]	Orbit 1 (planet)	Orbit 2 (binary)
a_i	[AU]	2.6 ± 0.1	21.00 ± 0.86
e_i		0.48 ± 0.02	0.42 ± 0.03
$_i$	[deg]	93.2 ± 3.0	241.9 ± 3.1
M_i	[deg]	349.1 ± 1.80	121 ± 45
Ω_i	[deg]	?	79.8 ± 0.1
I_i	[deg]	?	116.8 ± 0.7
m_i	[M_{Jup}]	$2.98/\sin I_1$	472

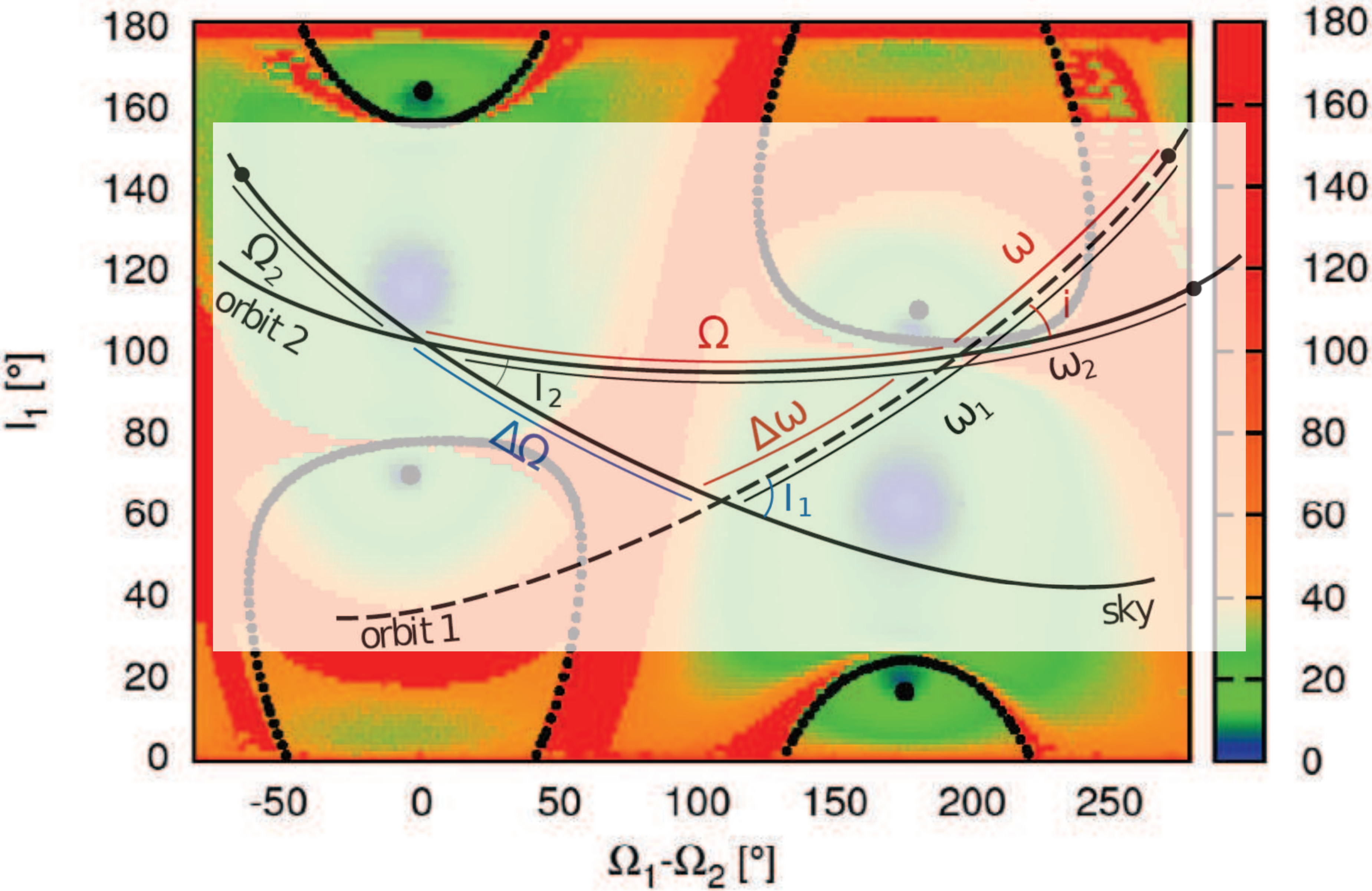
Correia et al. (A&A 2008), Chauvin et al. (A&A 2012)

HD 196885 b



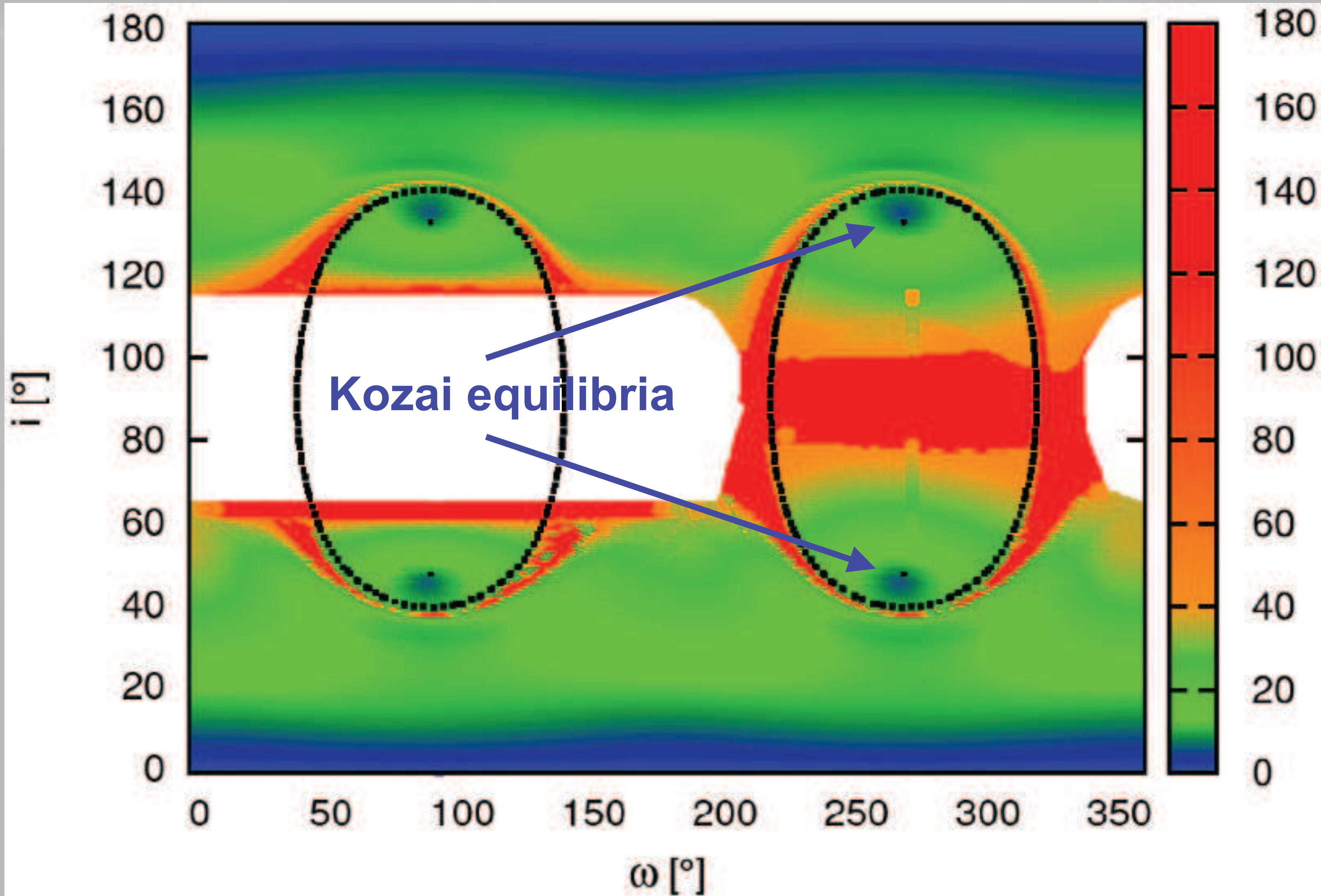
Giuppnne, Correia, et al. (A&A 2012)

HD 196885 b



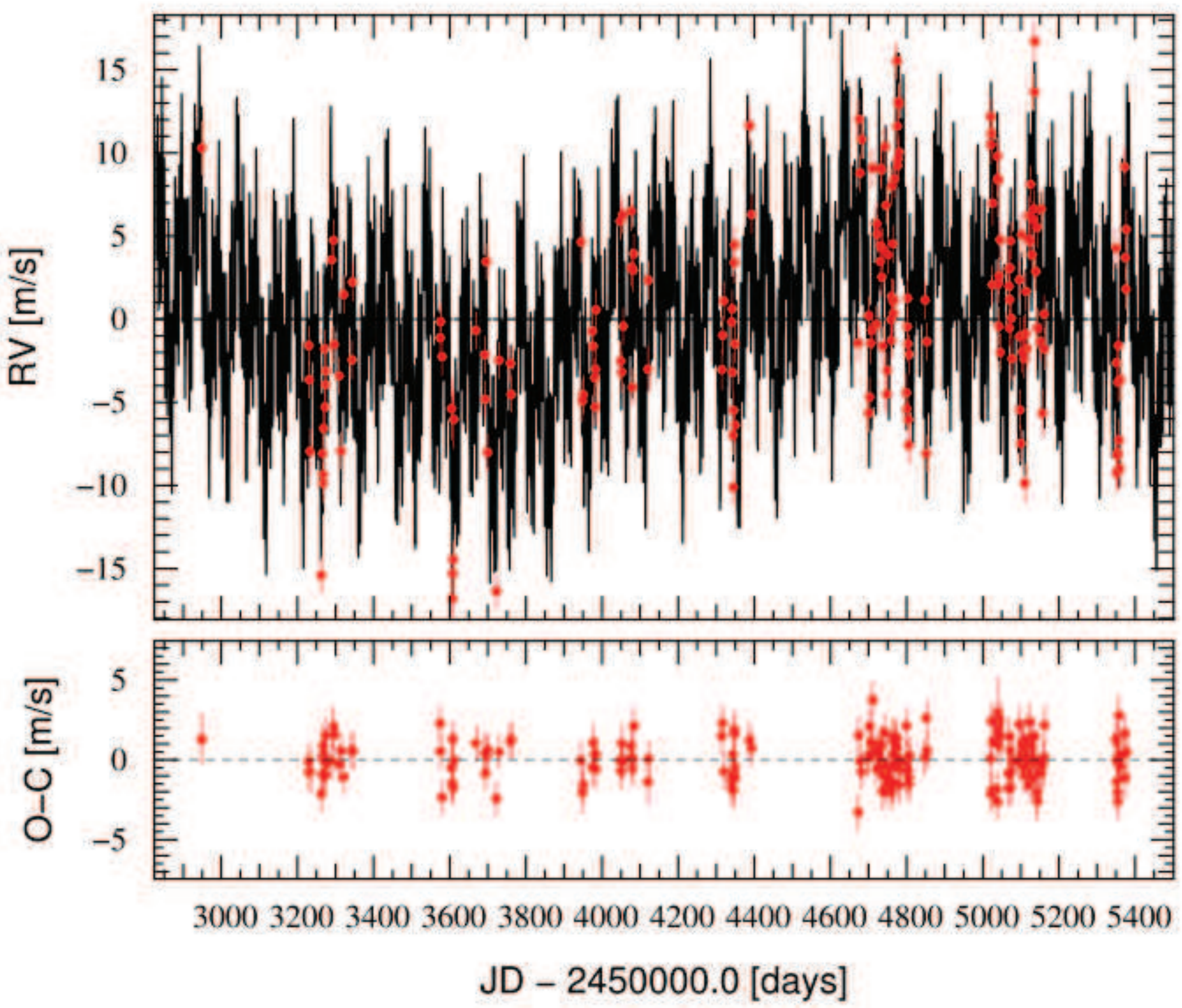
Giuppnne, Correia, et al. (A&A 2012)

HD 196885 b (Kozai planet?)



Giuponne, Correia, et al. (A&A 2012)

HD 10180, seven planets!



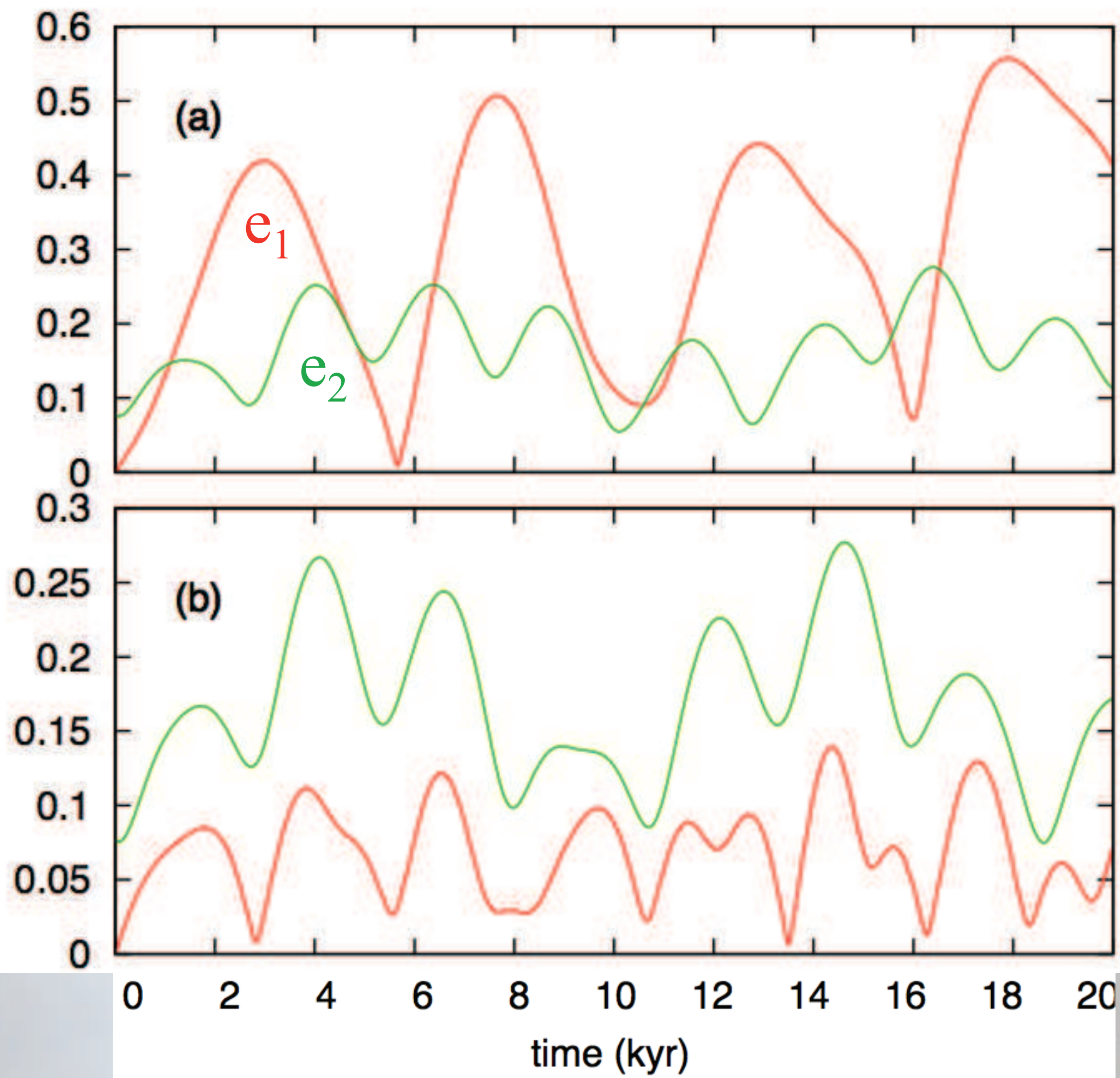
Lovis, Correia, Laskar et al. (A&A 2010)

keplerian orbital solution

Parameter	[unit]	HD 10180 b	HD 10180 c	HD 10180 d	HD 10180 e	HD 10180 f	HD 10180 g	HD 10180 h
Epoch	[BJD]				2,454,477.878676 (fixed)			
i	[deg]				90 (fixed)			
V	[km s^{-1}]				35.53014(±0.00045)			
P	[days]	1.177662 (±0.000090)	5.75962 (±0.00029)	16.3570 (±0.0042)	49.747 (±0.023)	122.72 (±0.19)	602 (±11)	2229 (±106)
	[deg]	142 (±11)	29.4 (±1.9)	99.4 (±3.3)	20.9 (±2.2)	237.8 (±3.2)	253 (±11)	317.6 (±4.1)
e		0.0 (fixed)	0.077 (±0.032)	0.142 (±0.060)	0.061 (±0.036)	0.127 (±0.066)	0.0 (fixed)	0.145 (±0.073)
	[deg]	0.0 (fixed)	−41 ($^{+70}_{-141}$)	−51 ($^{+43}_{-10}$)	171 (±60)	−37 ($^{+79}_{-209}$)	0.0 (fixed)	−166 (±58)
K	[m s^{-1}]	0.82 (±0.14)	4.53 (±0.15)	2.92 (±0.16)	4.26 (±0.18)	2.95 (±0.18)	1.55 (±0.22)	3.11 (±0.22)
$m \sin i$	[M]	1.40 (±0.25)	13.16 (±0.59)	11.91 (±0.75)	25.3 (±1.4)	23.5 (±1.7)	21.3 (±3.2)	65.2 (±4.6)
a	[AU]	0.02226 (±0.00038)	0.0641 (±0.0010)	0.1286 (±0.0021)	0.2695 (±0.0048)	0.4924 (±0.0083)	1.422 (±0.030)	3.40 (±0.12)
N_{meas}					190			
Span	[days]				2428			
rms	[m s^{-1}]				1.27			
$\sqrt{\chi^2}$					1.23			

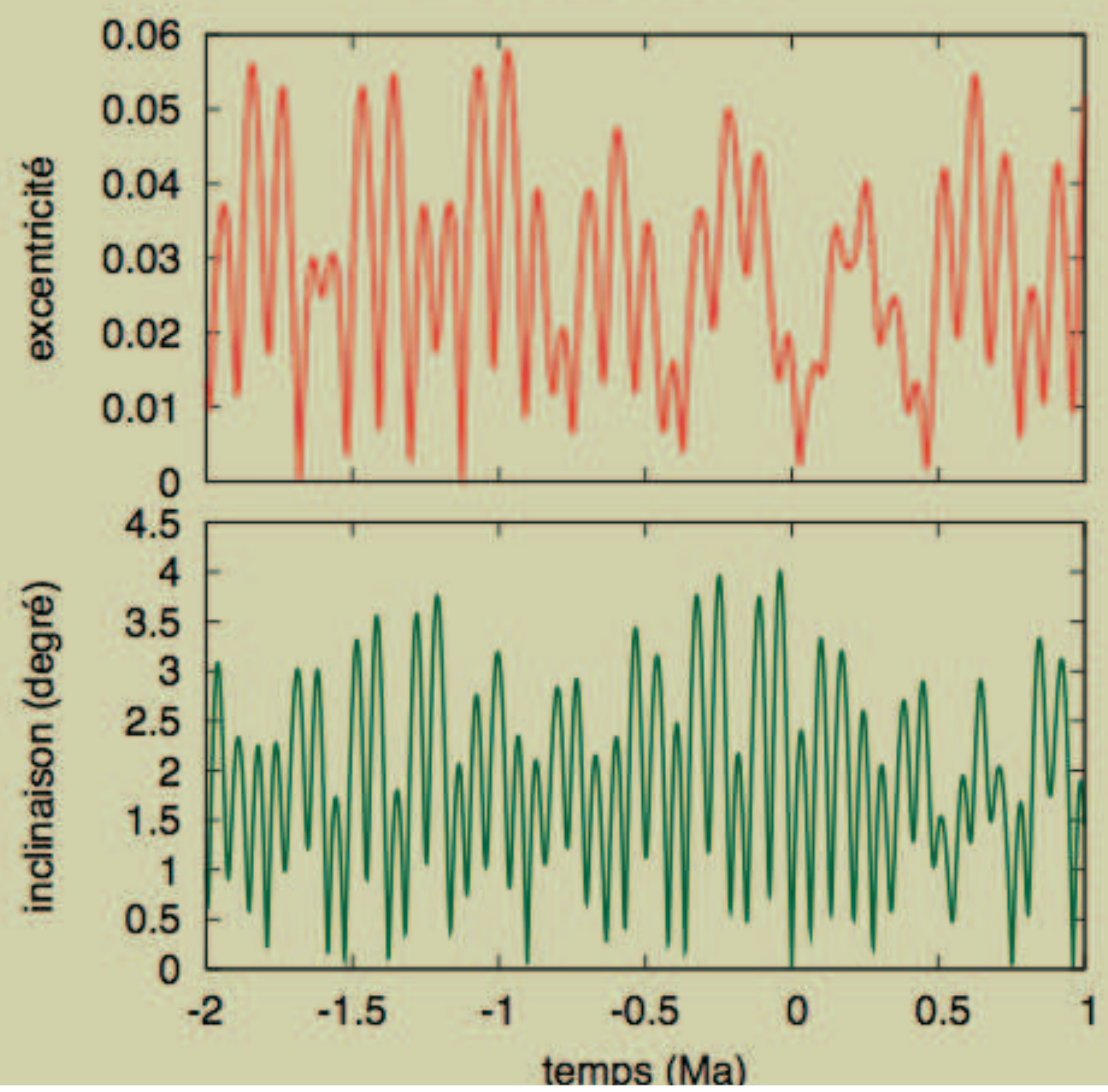
Lovis, Correia, Laskar et al. (A&A 2010)

unstable system?



Lovis, Correia, Laskar et al. (A&A 2010)

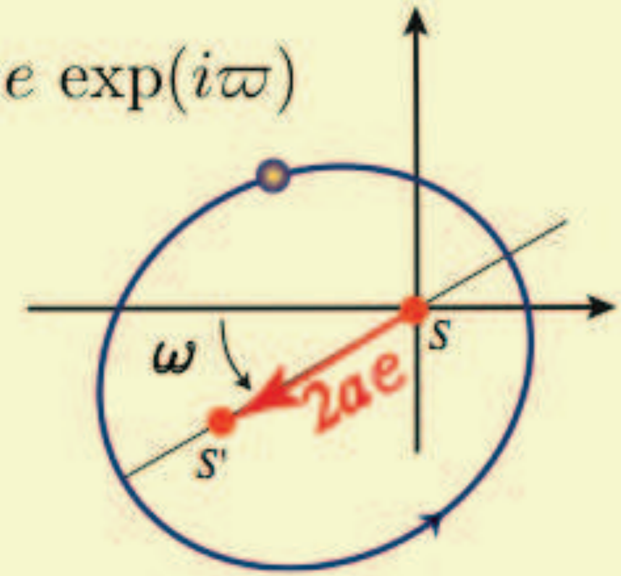
Secular variations of the orbital elements of the Earth



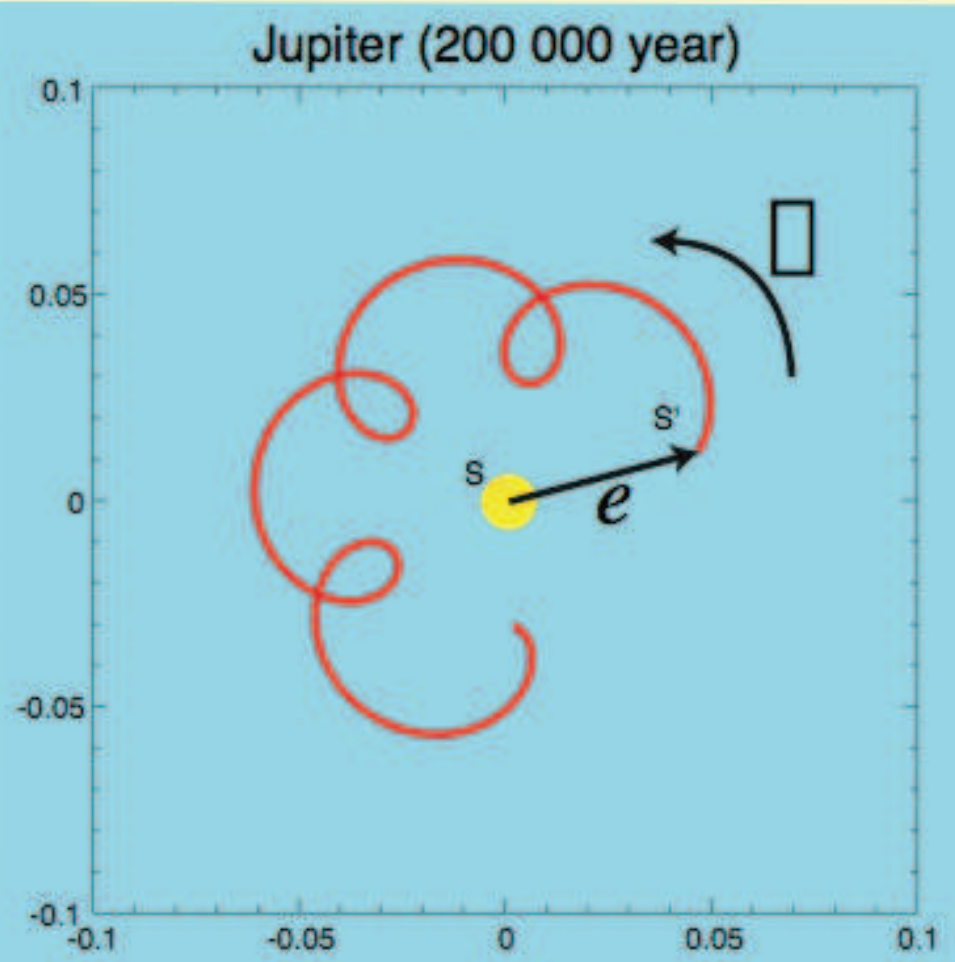
Secular variations of the eccentricities and inclinations

Lagrange 1774-78, Laplace 1774-75

$$z = e\ \exp(i\varpi)$$

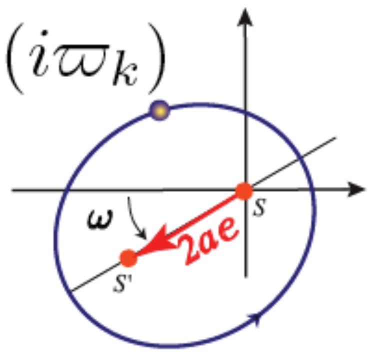


$$\frac{d}{dt}\begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_6 \end{pmatrix} = \sqrt{-1}\,A \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_6 \end{pmatrix}$$



Lagrange-Laplace linear system

$$z_k = e_k \exp(i\varpi_k)$$



$$\frac{d}{dt}\begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{pmatrix} = \sqrt{-1}\,A \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{pmatrix}$$

$$\begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{pmatrix} = (S) \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix}$$

$$z_k = \sum_j s_{kj} u_j$$

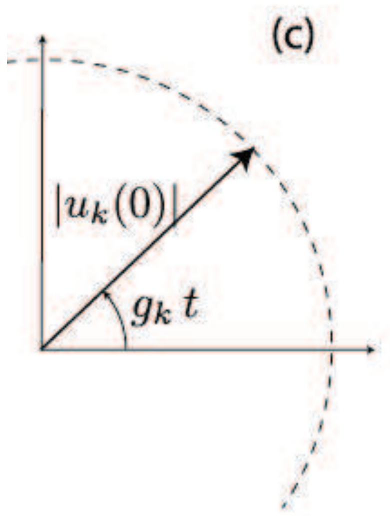
Diag = S^{-1} A S

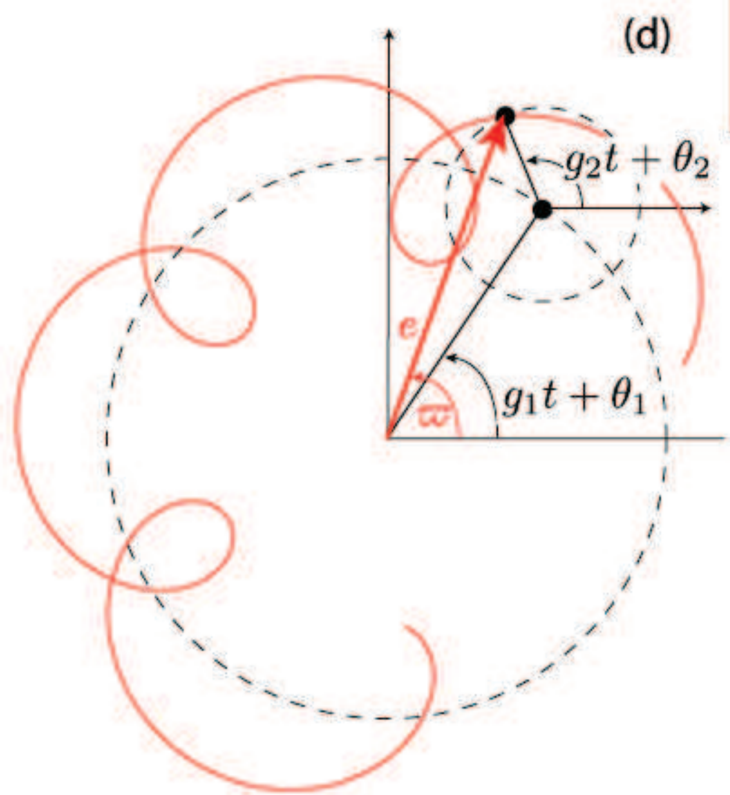
$$\frac{d}{dt}(u_k) = Diag(g_1, g_2, \ldots, g_n)(u_k)$$

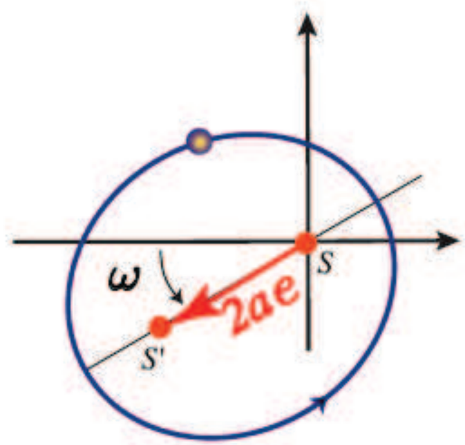
$$u_k = u_k(0)\exp(i g_k t)$$

$$\begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{pmatrix} = (S) \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix}$$

$$z_k = \sum_j s_{kj} u_j$$

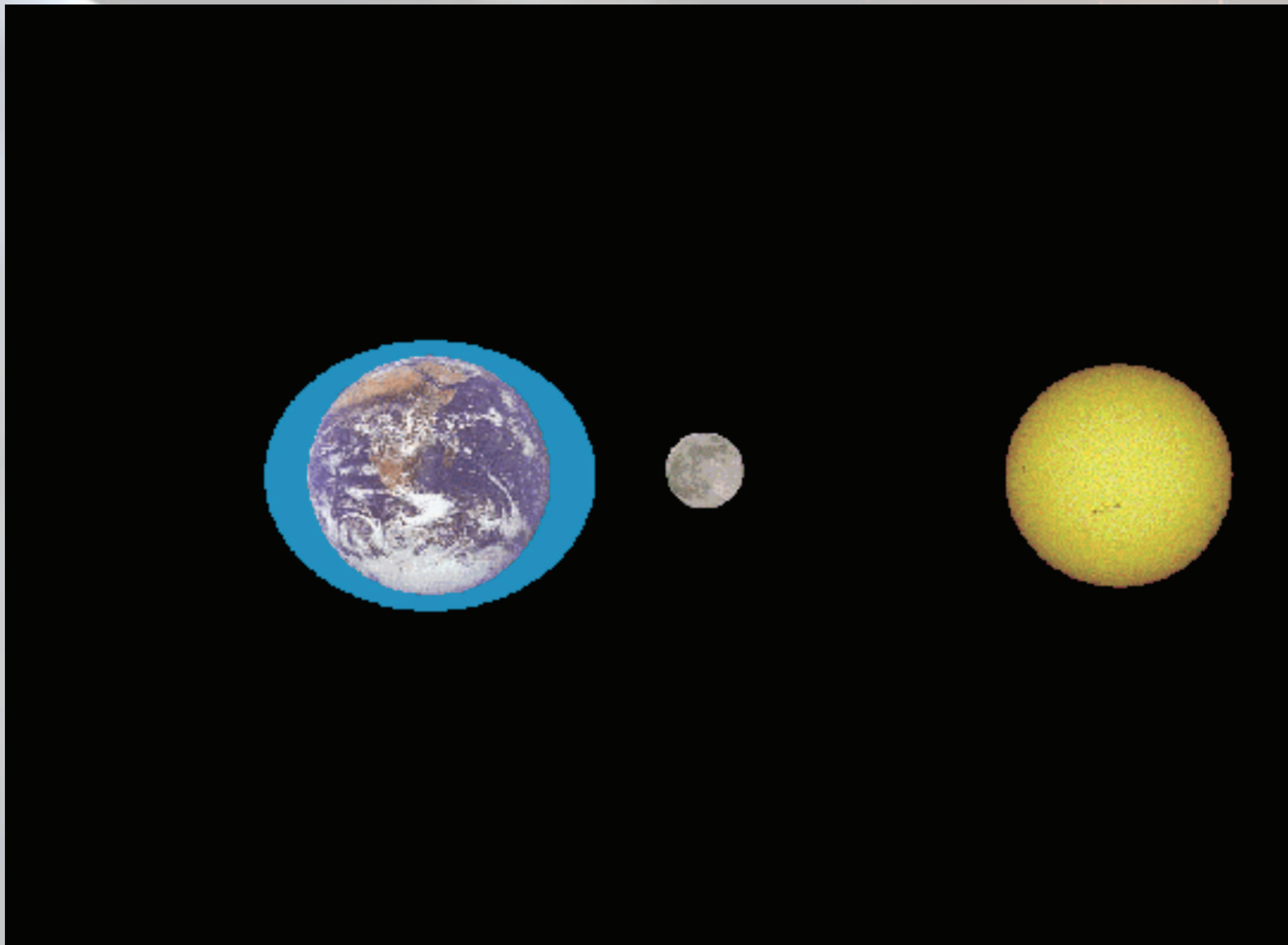






$$z_k = e_k \exp(i\varpi_k)$$

Tidal effects



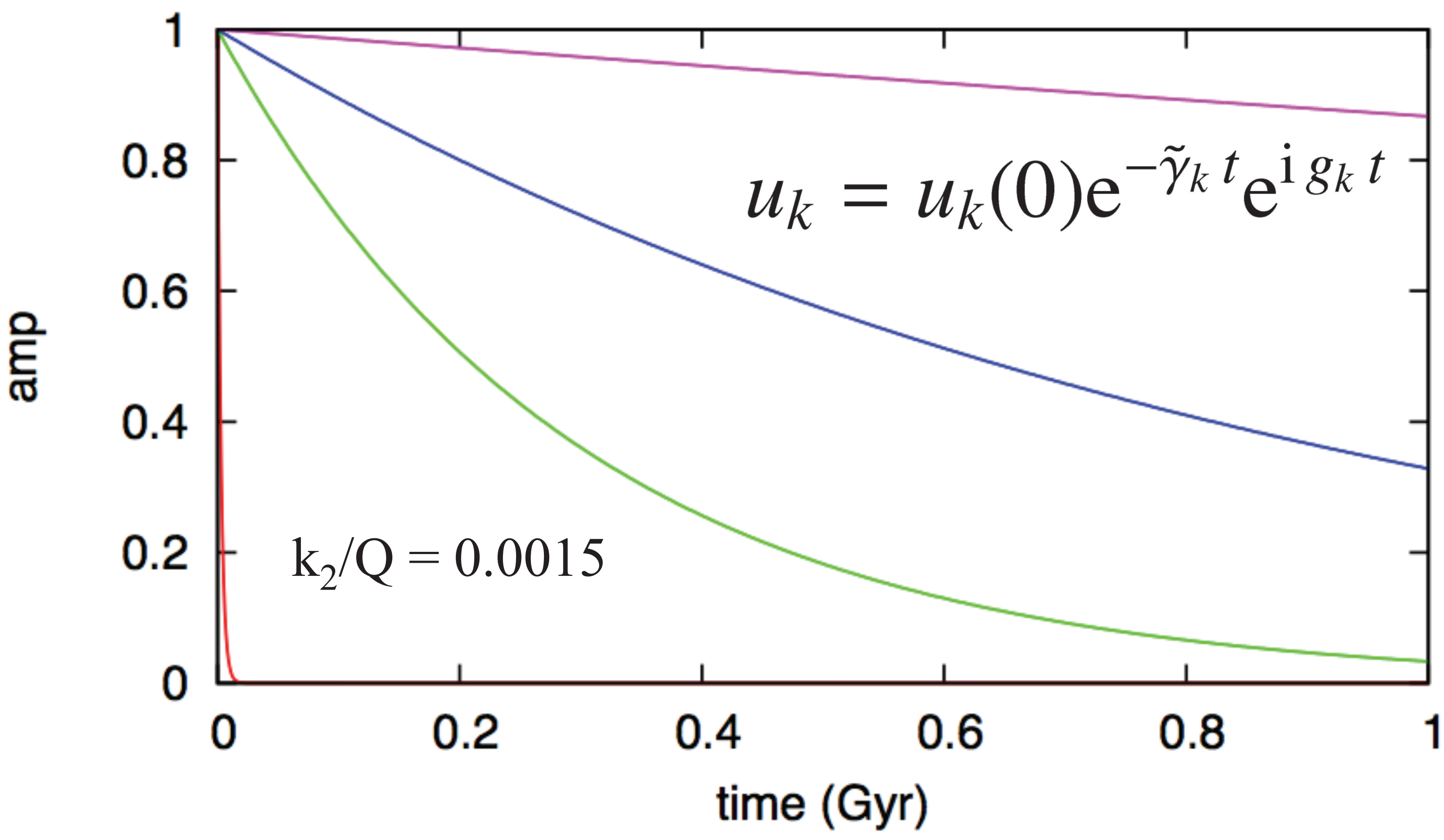
tidal dissipation

$$\dot{z}_k = -\gamma_k z_k$$

$$\gamma_k = K_k n_k$$

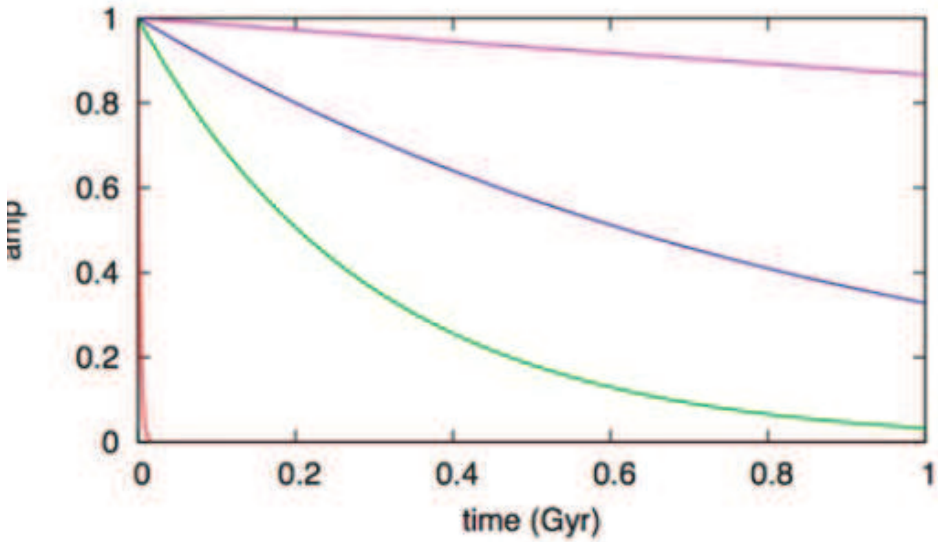
$$K = \frac{21}{2}\left(\frac{M}{m}\right)\left(\frac{R}{a}\right)^5 \frac{k_2}{Q}$$

tidal dissipation



Lovis, Correia, Laskar et al. (A&A 2010)

tidal constraint



$$u_k = u_k(0) e^{-\gamma_k t} e^{i g_k t}$$

1) compute an approximation of (S)

$$\begin{pmatrix} u_1 \\ \vdots \\ u_7 \end{pmatrix} \approx (S)^{-1} \begin{pmatrix} z_b \\ \vdots \\ z_h \end{pmatrix}$$

2) compute the u_k

$$u_k = \sum_j S^{-1}_{kj} z_j \approx 0$$

3) add the constraint to χ^2

$$\chi^2_R = R\left(u_1^2 + u_2^2\right)$$

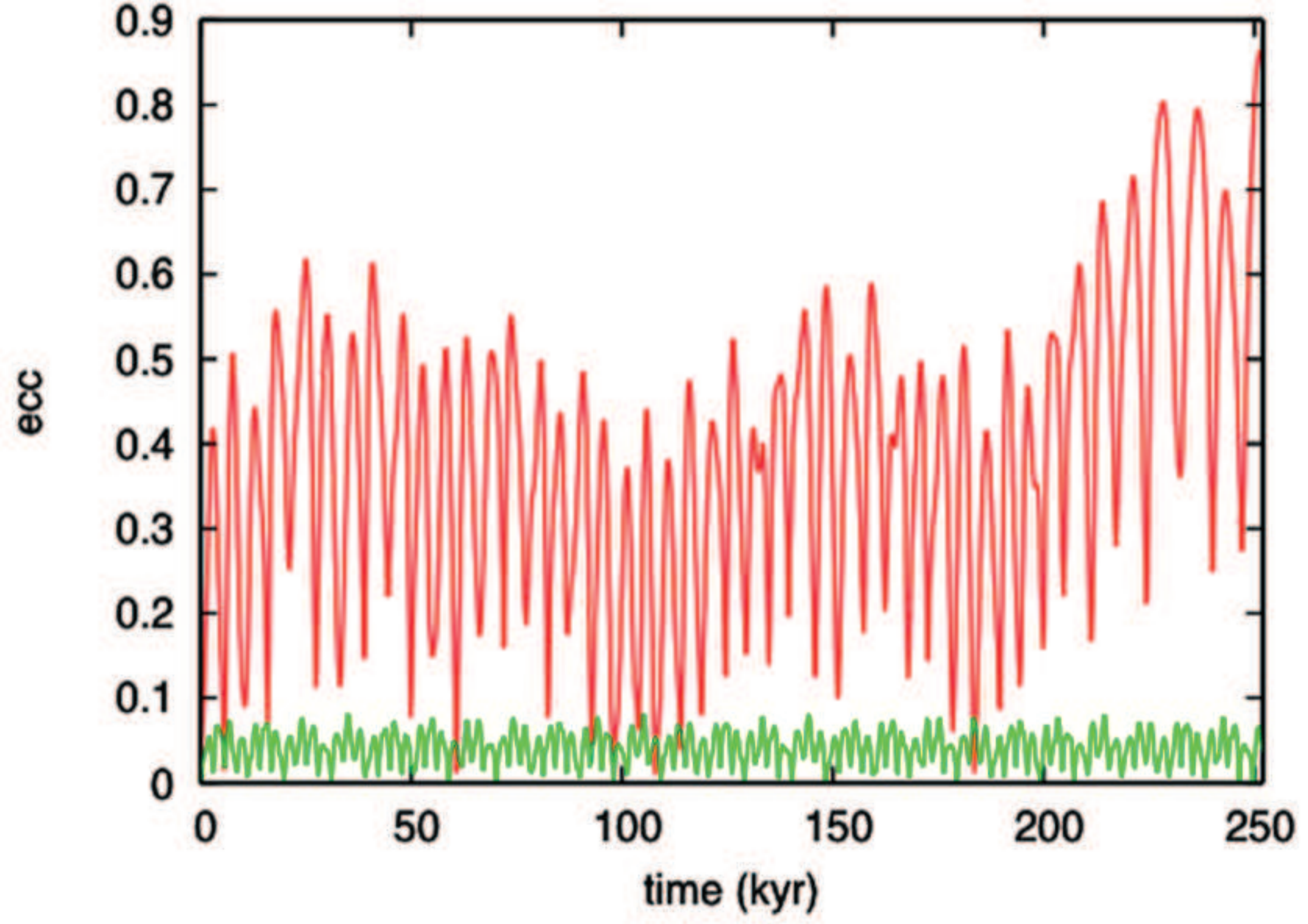
Lovis et al. (A&A 2010) Laskar, Boué, Correia, 2012

non-keplerian orbital solution, with tides

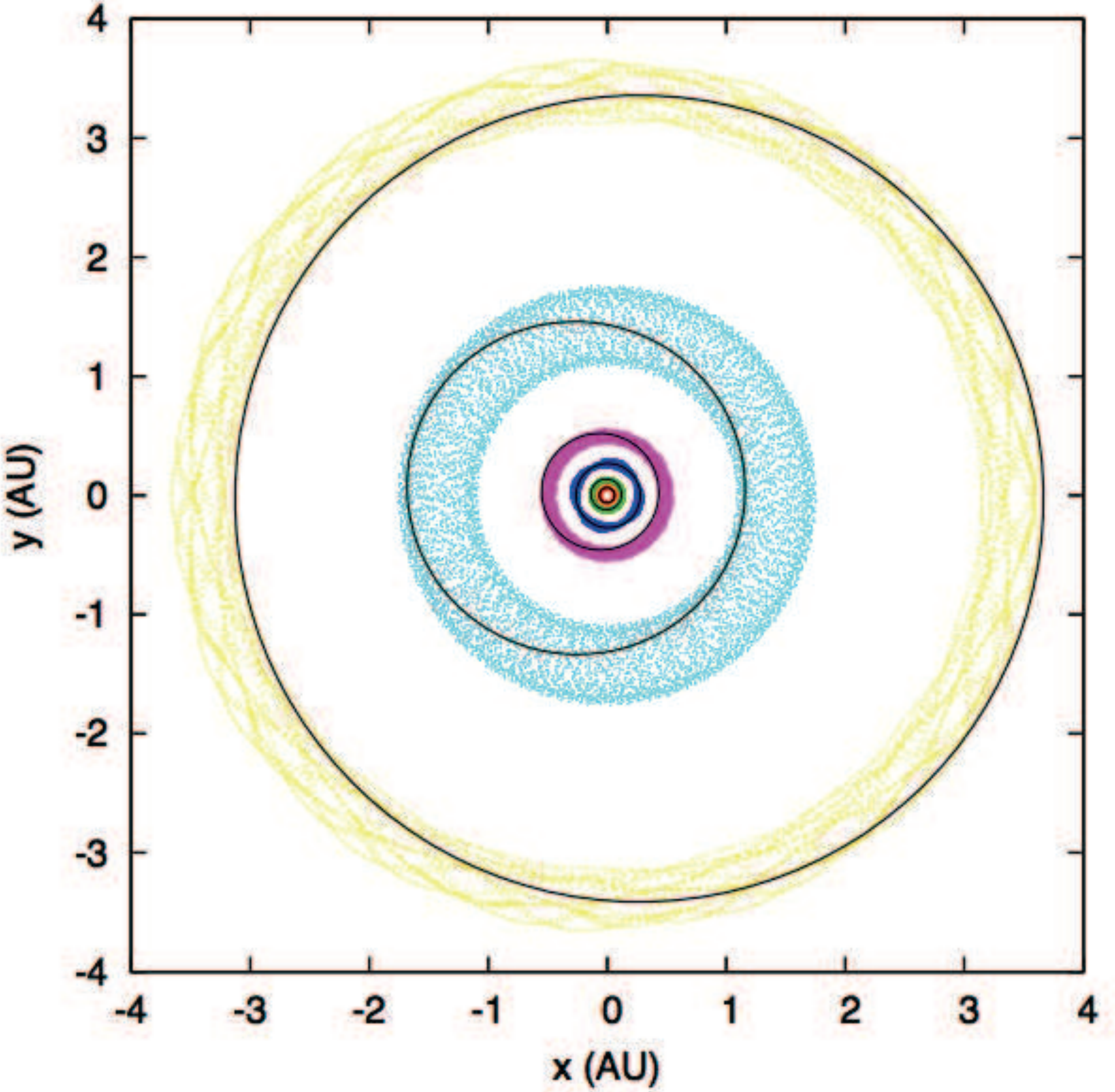
Parameter	[unit]	HD 10180 b	HD 10180 c	HD 10180 d	HD 10180 e	HD 10180 f	HD 10180 g	HD 10180 h
Epoch	[BJD]				2,454,000.0 (fixed)			
i	[deg]				90 (fixed)			
V	[km s^{-1}]				35.52981(±0.00012)			
P	[days]	1.17768 (±0.00010)	5.75979 (±0.00062)	16.3579 (±0.0038)	49.745 (±0.022)	122.76 (±0.17)	601.2 (±8.1)	2222 (±91)
	[deg]	188 (±13)	238.5 (±2.3)	196.6 (±3.8)	102.4 (±2.4)	251.2 (±3.6)	321.5 (±9.9)	235.7 (±6.0)
e		0.0000 (±0.0025)	0.045 (±0.026)	0.088 (±0.041)	0.026 (±0.036)	0.135 (±0.046)	0.19 (±0.14)	0.080 (±0.070)
	[deg]	39 (±78)	332 (±43)	315 (±33)	166 (±110)	332 (±20)	347 (±49)	174 (±74)
K	[m s^{-1}]	0.78 (±0.13)	4.50 (±0.12)	2.86 (±0.13)	4.19 (±0.14)	2.98 (±0.15)	1.59 (±0.25)	3.04 (±0.19)
$m \sin i$	[M]	1.35 (±0.23)	13.10 (±0.54)	11.75 (±0.65)	25.1 (±1.2)	23.9 (±1.4)	21.4 (±3.4)	64.4 (±4.6)
a	[AU]	0.02225 (±0.00035)	0.0641 (±0.0010)	0.1286 (±0.0020)	0.2699 (±0.0042)	0.4929 (±0.0078)	1.422 (±0.026)	3.40 (±0.11)
N_{meas}					190			
Span	[days]				2428			
rms	[m s^{-1}]				1.28			
2					1.24			

Lovis, Correia, Laskar et al. (A&A 2010)

eccentricity evolution



Stability over 10 Myr



Lovis, Correia, Laskar et al. (A&A 2010)

Conclusions:

- **Most of the time, a Keplerian fit is sufficient for the determination of the orbits. In all cases, a Keplerian fit is the first approximation.**
- **Multi-planet systems are very common, very interesting, but hard to disentangle from observational data.**
- **Better determinations of the orbital parameters of a system can be achieved when dynamical considerations are taken into account during the fitting procedure.**
- **For systems that appear to be unstable, specific studies need to be made. Up to now, the solution never simple.**
- **Radial velocities alone can fully determine the architecture of multi-planet systems without the input from astrometry or transits.**
- **Dynamical studies of these systems can help the observations when searching for additional planets in the system.**